A Long-Run Risks Model of Asset Pricing with Fat Tails*

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Abstract

We explore the effects of fat tails on the equilibrium implications of the long run risks model of asset pricing by introducing innovations with dampened power law to consumption and dividends growth processes. We estimate the structural parameters of the proposed model by maximum likelihood. We find that the stochastic volatility model with fat tails can generate implied risk premium, expected risk free rate and their volatilities comparable to the magnitudes observed in data. The model with fat tails leads to a significant increase in implied risk premia over the benchmark Gaussian model, but similar risk free rate and its volatility, as well as that of market returns and price-dividend ratios.

Keywords: asset pricing, long run risks, equity risk premium, fat tails, Dampened Power Law, Lévy process, stochastic volatility

JEL classification: G12, G13, E43

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1 Introduction

The long-run risks model of asset pricing, developed by Bansal and Yaron (2004), provides sound theoretical rationalization for several empirical characteristics of financial markets, such as market risk premium and asset return volatilities. Their model features a long-run risk component, along with stochastic volatility, in consumption and dividend growth processes in a conditionally Gaussian world. Essentially, in this framework, risk-averse agents demand higher equity premium due to persistent effects of the long-run risk component. Bansal (2007) provides a comprehensive review of the long-run risks model.

The presence of fat tails would result in agents with risk aversion demanding higher equity premium than in a Gaussian world, since fat tails imply more frequent occurrence of extreme events. Many financial and macroeconomic time series exhibit fat tails.[1] One could ask how much fat tails would increase the magnitude of implied risk premium in a long-run risks model of Bansal and Yaron (2004) under reasonable assumptions about agents' preferences. We attempt to provide a quantitative assessment of a long-run risks model with fat tails in order to answer this question.

Several papers attempt to document the asset pricing implications of fat tails. Bidarkota and Dupoyet (2007) report that the introduction of fat tails to consumption growth process produces 80% higher risk premium compared to a lognormal process. However, their model does not feature long run risks or recursive utility as in Bansal and Yaron (2004). Shaliastovich and Tauchen (2008) assume that non-normality of consumption and dividend growth comes from a Lévy innovation to an AR(1) economy-wide state variable. This time-varying state variable time-changes both consumption and dividend growth. As in Bansal and Yaron (2004), they assume a utility function of the Epstein and Zin (1989) type. They calibrate the structural parameters of their model and find that their model can generate 4.5% implied

[1] Mandelbrot (1963) and Fama (1965) are the early studies documenting fat tails in financial time series. Cont and Tankov (2004) is an excellent exposition on financial modeling under non-Gaussian settings. Blanchard and Watson (1986), Balke and Fomby (1994) and Kiani and Bidarkota (2004) provide empirical evidence on the presence of fat tails in macroeconomic data.

risk premium but only with a very high risk aversion coefficient of 50. By contrast, Bansal and Yaron (2004) are able to generate 6.8% equity risk premium with a risk aversion of 10 assuming stochastic volatility in the consumption and dividend processes. Eraker and Shaliastovich (2008) model volatility of consumption growth as a mean-reverting Gamma-jump process that can accommodate fat tails. They focus on option pricing implications of their model, although they do provide a solution to general asset prices.

Bidarkota, Dupoyet and McCulloch (2007) explore the effects of non-normality on asset pricing through α-stable process under incomplete information. By imposing restrictions on the parameters of the stable distribution, they guarantee finiteness of relevant moments of interest necessary for asset pricing. They generate volatility persistence of implied returns of a magnitude comparable to that in the data. However, their implied risk premium is 4%, well shy of the over-6% value observed in the data. Martin (2008) considers the impact of higher moments of consumption growth process on asset pricing, but without imposing long-run risks. His model captures empirical features more general than fat tails in consumption and dividend growth process by utilizing the cumulant generating function of non-normal processes.

We also consider the effects of stochastic volatility although the main focus of this study is a quantitative evaluation of the effects of fat tails. We only model stochastic volatility in the long-run risk component with a mean-reverting square-root process. This type of affine process is also recently adopted by Tauchen (2005), Bansal and Shaliastovich (2008), Drechsler and Yaron (2008) in the asset pricing literature.

In this paper, we choose a succinct long-run risk (LRR) model to account for possible fat tails and stochastic volatilities in the consumption and dividends growth processes. Fat tails are modeled as a dampened power law (DPL) process, as in Wu (2006b). Stochastic volatility is modeled in the innovations to the long-run risk component. The representative agent's preferences are assumed to be of Epstein and Zin (1989) recursive type. Drechsler and Yaron (2008) allow for both features by introducing stochastic volatilities (diffusion) in consumption and dividend growth and compound Poisson jumps in the long-run risk and stochastic volatility, but their main focus is on variance risk premia. The main differences between their study and ours are two-fold: (1) we directly model jumps in consumption and dividend growth, but not in long-run risks[2]; (2) our DPL jump processes can allow for both finite/infinite jump activities, and finite/infinite variation[3], while Poisson-type jump is a finite activity jump (or a large and rare event). Our choice of LRR model is amenable to maximum likelihood estimation.

With this model framework, we first estimate all structural parameters, including persistence of the long run component, via maximum likelihood. We then evaluate the model-implied risk premium and the risk free rate, and their volatilities with the estimated values of the structural parameters. Using quarterly consumption, dividends and confidence data spanning the period from 1968 through 2007[4], we find that our model with fat tails can generate about 3.54% expected market risk premium and 1.67 % expected risk free rate with the magnitudes of risk aversion and elasticity of intertemporal substitution being 17.5 and 1.5, respectively. These values are significantly better than what the benchmark Gaussian model can produce (-0.03% equity risk premium and 1.84% risk free rate). We also show that the model with fat tails generates higher volatility of price-dividend ratios. Using an alternative method for estimating the long-run risk component, we report more impressive empirical results, in which expected market risk premium and risk free rate for the fat-tailed model are 5.50% and 1.25%[5](comparable to observed values in the data) compared to 3.98% and 2.05% for the benchmark Gaussian model with the magnitudes of risk aversion and

[2]We considered jumps in long-run risks in an earlier study, but econometric estimation shows that such a model exhibits a worse match to actual data.

[3]A DPL-type jump can be a small or large, frequent or infrequent, event. This flexible specification allows the data to determine the actual jump process, without restricting the type of jump a priori. Small and frequent jumps resemble diffusion with respect to their sample paths.

[4]We have to restrict our study to data after 1968, since confidence data are only available after the fourth quarter of 1968. In fact, using only consumption and dividend data from 1947 to 2007, we document that our model with fat tails (but without stochastic volatility) produces a significantly higher market risk premium than a Gaussian model in an earlier version of this paper.

[5]Drechsler and Yaron's (2008) calibrated model also generates an impressive results matching actual data, but with risk aversion being 10 (significantly higher than ours with the alternative estimation method).

elasticity of intertemporal substitution being 4 and 1.5, respectively. In both frameworks, the fat-tailed model exhibits a marked improvement over the Gaussian model.

The paper is organized as follows. Section 2 introduces the model with long run risks and fat tails, and summarizes the solutions to asset prices in such a setting. Section 3 presents data, discusses estimation methodology, and reports maximum likelihood model estimation results. Section 4 analyzes the asset pricing implications. Section 5 concludes with a brief summary of the main implications of modeling fat tails with long run risks and recursive utility.

2 Model

We begin with a description of the pricing kernel in a long-run risks model in subsection 2.1 and then propose a consumption growth process with fat tails in subsection 2.2. This is a modification of Bansal and Yaron's (2004) model. We then derive the asset pricing implications under our consumption growth process in the last subsection.

2.1 Pricing Kernel

A representative agent in the economy exhibits recursive preferences as in Epstein and Zin (1989) and Weil (1989). The single period utility separates risk aversion and intertemporal elasticity of substitution in the following form:

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\}^{\frac{\theta}{1-\gamma}} \tag{1}$$

where the parameters δ, γ and ψ are the time discount factor, the risk aversion coefficient and the intertemporal elasticity of substitution (IES), respectively. The parameter θ is defined by $\frac{1-\gamma}{1-\frac{1}{\psi}}$.

The representative agent faces the following first-order condition, or the Euler's equation:

$$E_t[\delta^\theta G_{t+1}^{\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \tag{2}$$

where $R_{i,t+1}$, $R_{a,t+1}$, and G_{t+1} are the gross returns on any asset i, the gross returns on the aggregate consumption portfolio, and the gross growth rate of consumption, respectively. The aggregate consumption portfolio pays aggregate consumption as its dividend every period. $M_{t+1} = \delta^\theta G_{t+1}^{\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)}$ is often called the "Intertemporal Marginal Rate of Substitution" (IMRS) or the pricing kernel, which applies to any asset return $R_{i,t+1}$ in the economy. In order to price any individual asset, we alternatively replace $R_{i,t+1}$ in the above equation with either the aggregate consumption portfolio returns $R_{a,t+1}$, or with the market portfolio returns $R_{m,t+1}$ that pay the aggregate market dividend, or with the risk free asset returns $R_{f,t+1}$ that pay one unit of consumption good as dividends every period.

We use the following notation in the rest of the paper:

$$r_{i,t+1} = lnR_{i,t+1}$$

$$r_{a,t+1} = lnR_{a,t+1} = ln\frac{P_{a,t+1} + C_{a,t+1}}{P_{a,t+1}} \tag{3}$$

$$r_{m,t+1} = lnR_{m,t+1} = ln\frac{P_{m,t+1} + D_{t+1}}{P_{m,t+1}} \tag{4}$$

$$r_{f,t+1} = lnR_{f,t+1}$$

$$m_{t+1} = lnM_{t+1} = \theta ln\delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{5}$$

where $P_{a,t+1}$ and $P_{m,t+1}$ are the prices of aggregate consumption and market portfolios, respectively. We drop the subscript "a" in aggregate consumption $C_{a,t+1}$ in the rest of the paper.

The definitions of $r_{a,t+1}$ and $r_{m,t+1}$ in Equations (3) and (4) reflect the fact that the consumption portfolio pays aggregate consumption C_{t+1} as its dividend whereas the market portfolio pays out D_{t+1}. We can relate the prices of consumption and market portfolios to price-dividend ratios of these two assets, namely $z_t = ln\frac{P_{a,t}}{C_t}$ and $z_{m,t} = ln\frac{P_t}{D_t}$. Using their definitions, we expand the aggregate and market returns by Taylor's expansion around the mean of z_t and $z_{m,t}$ respectively as in Campbell and Shiller (1988) to obtain:

$$r_{a,t+1} \simeq k_0 + k_1 z_{t+1} - z_t + g_{c,t+1} \tag{6}$$

$$r_{m,t+1} \simeq k_{0m} + k_{1m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \tag{7}$$

where $g_{c,t+1} = ln\frac{C_{t+1}}{C_t}$ and $g_{d,t+1} = ln\frac{D_{t+1}}{D_t}$ are the consumption and dividends growth rates. We complete our model specification by specifying the dynamics of consumption and dividends growth rates in the following section.

2.2 Dynamics of Consumption and Dividends Growth Rates

We first specify the benchmark model - one in which all shocks to consumption and dividend growth rates processes are Gaussian:

$$g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1} \tag{8}$$

$$x_{t+1} = \rho x_t + \sigma_e \sqrt{v_t} e_{t+1} \tag{9}$$

$$v_{t+1} = (1-l)\bar{v} + l v_t + \sigma_v \sqrt{v_t} \epsilon_{t+1} \tag{10}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1} \tag{11}$$

where $\eta_{c,t} \sim iidN(0, \sigma_c^2)$, $\eta_{d,t} \sim iidN(0, \sigma_d^2)$, $e_t \sim iidN(0,1)$ and $\epsilon_t \sim iidN(0,1)$.

In this model, both consumption and dividend growth rates are made up of a non-zero constant mean, a persistent component x_t, and noise. This process is similar to the Gaussian fluctuating-uncertainty model of Bansal and Yaron (2004). Equation 10 generates time-varying stochastic volatility v_t. We call v_t a confidence measure, as proposed and estimated by Bansal and Shaliastovich (2008) using Survey of Professional Forecasters data from Federal Reserve Philadelphia. As in Bansal and Yaron (2004), we assume that agents observe the persistent component and set equilibrium asset prices accordingly.

For the more general conditionally non-Gaussian model, we consider an alternative growth rates process that features non-normality based on the well-documented evidence of fat tails in macroeconomic (including consumption) data (see footnote 1 for references), As reported subsequently in Section 3, the data also show deviation of dividend growth rates from normality. Therefore we allow the innovations to consumption and dividend growth rates $\eta_{c,t+1}$ and $\eta_{d,t+1}$ to follow a fat-tailed distribution.

As noted in Geweke (2001), we often encounter difficulty in ensuring finiteness of exponential moments of a fat-tailed distribution. This is essential for ensuring finiteness of asset prices in these kinds of models. One approach to overcoming this difficulty is to use "dampened power law" (henceforth DPL) process as in Wu (2006b) to model fat tails. See also Cont and Tankov (2004) and Shaliastovich and Tauchen (2008). An advantage of this approach is tractability when we apply Fourier transform to derive the cumulant generating (and characteristic) function that appears in asset pricing formulae as seen in the following section.

We refer to our model with fat tails as "the DPL model":

$$g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1} \tag{12}$$

$$x_{t+1} = \rho x_t + \sigma_e \sqrt{v_t} e_{t+1} \tag{13}$$

$$v_{t+1} = (1-l)\bar{v} + l v_t + \sigma_v \sqrt{v_t} \epsilon_{t+1} \tag{14}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1} \tag{15}$$

where e_t and ϵ_t follow $iidN(0,1)$, $\eta_{c,t}$ and $\eta_{d,t}$ obey two independent DPL processes. The two DPL process are defined by their Lévy densities $\nu(\eta)$:

$$\nu(\eta) = \begin{cases} \gamma_+ e^{-\beta_+|\eta|}|\eta|^{-\alpha-1}, & \eta > 0 \\ \gamma_- e^{-\beta_-|\eta|}|\eta|^{-\alpha-1}, & \eta < 0. \end{cases}$$

This specification allows for leptokurtosis and skewness in innovations to consumption and dividends growth rates. The former is controlled by α, while the latter arises from the asymmetry of the scale parameters γ_+ and γ_- and the dampening parameters β_+ and β_-. A DPL process without dampening, i.e. with $\beta_+ = \beta_- = 0$, becomes an $\alpha-$stable distribution. Hence, dampened power law is also called a "tempered stable" distribution. DPL process was used in consumption-based asset pricing by Bidarkota and Dupoyet (2007). DPL distribution, without dampening and with $\alpha = 2$, results in the Gaussian distribution.

Both the Gaussian and DPL models presented in the subsection feature stochastic volatility. These two models degenerate to iid models by assuming constant v_t and $\epsilon_t = 0$. The two categories of models are called " SV Model" and "iid Model" in the rest of the paper.

2.3 Equilibrium

With the specification of exogenous consumption and dividend growth rates, we can proceed with deriving the pricing kernel m_t, returns on the aggregate consumption $r_{a,t}$, the risk-free rate $r_{f,t}$, the market return $r_{m,t}$, and volatilities of asset returns. The key to deriving all these quantities are the log price-dividend ratios z_t and $z_{m,t}$ on the consumption and market portfolios. The linear specification of the growth dynamics guarantees concise solutions to both ratios. In the following, we discuss the solution method to the DPL model in some detail.

Observing that there are two state variables x_t and v_t in the economy, we conjecture that log price-consumption ratio z_t and log price-dividend ratio $z_{m,t}$ in the DPL model take the following form $z_t = b_0 + b_x x_t + b_v v_t$ and $z_{m,t} = b_{0m} + b_{xm} x_t + b_{vm} v_t$. Based on this conjecture, we solve for z_t and $z_{m,t}$ by substituting them in Euler's equation. The derivations of individual returns, namely aggregate return on the consumption portfolio $r_{a,t+1}$, risk free return $r_{f,t+1}$, and the market return $r_{m,t+1}$ involve the cumulant exponent of Lévy process. Risk premia and variance of respective returns can then be easily obtained. Detailed derivation for both DPL and Gaussian models are available in Appendices A and B. Here, we only summarize the main results and briefly discuss the dependence of these results on the persistence of the long run component ρ, the variances of innovation to the long run component σ_e^2 and dividend growth $Var(\eta_d)$.

The price-consumption and price-dividend ratios z_t and $z_{m,t}$ are derived in Appendix A.1 and A.2. The unconditional variance of the market price-dividend ratio is $Var(z_{m,t}) = b_{xm}^2 Var(x_t) + b_{vm}^2 Var(v_t) = \frac{b_{xm}^2 \bar{v}}{1-\rho^2}\sigma_e^2 + \frac{b_{vm}^2 \bar{v}}{1-l^2}\sigma_v^2$, where $\bar{v}$ is unconditional expectation of confidence measure. Examining the formula reveals that $Var(z_{m,t})$ is positively dependent on the persistence (ρ), the speed of mean-reversion (l) and the variances of innovation to the long run component (σ_e^2) and confidence measure (σ_v^2).

Returns on the aggregate consumption portfolio are derived as Equation (A18) in Appendix A.3.

The pricing kernel (IMRS) m_{t+1} is derived in Appendix A.4. The unconditional variance of the pricing kernel $Var(m_{t+1})$ is given by Equation (A22). $Var(m_{t+1})$ is determined by the variances of the two state variables $Var(x_t)$ and $Var(v_t)$, variances of innovations to the two state variables, and the second moment of the innovation to the DPL consumption growth rates.

The expected risk free rate $E(r_{f,t+1})$ is derived as Equation (A24) in Appendix A.5. $E(r_{f,t+1})$ is determined by non-time-varying mean component of consumption growth μ_c, the variances of innovation to the long run component σ_e^2 and confidence measure σ_v^2, and cumulant exponent of the DPL component of consumption growth.

The market return and the market risk premium are given by Equations (A26) in Appendix A.6 and (A27) respectively. The market risk premium $E[r_{m,t+1} - r_{f,t}]$ is mainly determined by variances of innovations to all four processes underlying consumption and dividend growth rates, namely σ_e^2, σ_v^2, and two cumulant exponents of the DPL innovation.

The conditional and unconditional variances of market return are given by Equations (A28) and (A29). The unconditional variance is determined by the variances of the innovations to the two state variables σ_e^2, σ_v^2 and dividend growth $Var(\eta_d)$.

3 Data and Estimation

This section presents details on the data used, discusses estimation of consumption and dividends growth processes, as well as of confidence dynamics, and reports their maximum likelihood estimates. Hypotheses tests are also conducted to narrow down a best-fitting model incorporating fat tails.

3.1 Data Description

We employ quarterly US real consumption data on non-durables and services, US real dividends and Survey of Professional Forecasters (SPF) data from the fourth quarter of 1968 through the fourth quarter of 2007. Consumption data are obtained from the National Income and Product Accounts (NIPA) tables published by the Bureau of Economic Analysis (BEA). Consumer Price Indices (CPI) used to construct real values are obtained from the Bureau of Labor Statistics (BLS) publications. We aggregate monthly dividends data obtained from Robert Shiller's website to quarterly frequency.[6] Dividends are paid toward the S&P 500 index. Quarterly Survey of Professional Forecasters data (1968:IV–2007:IV) are available at Federal Reserve Philadelphia website. We follow Bansal and Shalistovich's (2008) method to construct confidence measure, namely the cross-sectional variance of the average forecast in the data.[7] Table 1 presents summary statistics for the data and Figure 1 plots the consumption and dividends growth rates.

Annualized standard deviation of consumption growth is 0.0118 during the period 1968-2007, compared to 0.0293 in Bansal and Yaron (2004) (for the period 1929-1998), 0.0357 in Mehra and Prescott (1985) (for the period 1889-1978), and 0.03226 in Bidarkota and Dupoyet (2007) (for the period 1889-1997). Since we use essentially the same source of consumption data as these other studies, the difference arises solely from differing sample

[6] `http://www.econ.yale.edu/~shiller/data.htm`

[7] Confidence measure is computed as $v_t = \frac{1}{n_t} Var(log(\frac{RGDP_{i,t}}{RGDP_t}))$, where n_t, $RGDP_{i,t}$ and $RGDP_t$ are the number of forecasters, the next quarter forecast of real GDP by forecaster i and real GDP during each survey period. Real GDP (forecast) is nominal GDP (forecast) deflated by price index (forecast).

periods used. Clearly, post-war consumption is much less volatile than that dating back to 1929 or 1889.

Dividends growth rates are more volatile than consumption growth rates. Annualized standard deviation of dividends growth rates is 0.0252 in our sample, compared to 0.115 in Bansal and Yaron (2004), and 0.112 in Campbell (1999) (for the period 1947-1995). The latter two studies use dividends to the CRSP value-weighted NYSE stock index. Differences in summary statistics of consumption and dividend growth rates between our data sample and these other studies have significant implications for asset pricing that we will examine in the next section.

Jarque-Bera tests reported in Table 1 show that consumption and dividend growth rates exhibit highly significant and somewhat lesser degrees of non-normality, respectively. Based on this observation, we consider model specification in Equations (12-15), namely that non-Gaussian (fat-tailed) shocks drive both consumption and dividend growth rates.

3.2 Model Estimation

Agents are assumed to observe v_t and x_t in Equations (8-11) and (12-15). Values of the confidence measure v_t are computed as described in the previous section. Parameter estimates for Equation (10 or 14) are obtained through Maximum Likelihood. However, since we (econometricians) do not have data on x_t, we estimate Equations (8-9) and (12-13) as unobserved components models, and use the resulting filtered mean of x_t as "the data" on x_t that investors are assumed to observe in setting equilibrium asset prices. Estimation of the unobserved components models involves either Kalman filtering in the fully Gaussian model of Equations (8-9), or the more general Sorenson and Alspach (1971) filter in the DPL model of Equations (12-13). In order to avoid complications resulting from bivariate observation equations (8, 11) and (12, 15), especially for the non-Gaussian model, we simplify by ignoring dividends data while estimating the long run risks component x_t. Thus, we estimate Equations (8-9) and (12-13), obtain filtered mean of x_t, and use these values to run regressions in Equations (11) and (15). To check robustness of our results, however, we also reverse the roles of consumption and dividends data in model estimation. We report results for this latter case in subsection 4.4.

In estimating the DPL model, we employ a Bayesian filtering technique proposed by Sorenson and Alspach (1971), which boils down to the Kalman filter under Gaussian innovations, but unlike the latter, is efficient under non-Gaussian innovations as well. The following describes the filtering procedure using consumption process as the observation equation. Denote $G_{c,t}$ as the history of consumption growth up to time t, comprising of $g_{c,1}, g_{c,2}, ..., g_{c,t}$. The predictive and filtering densities of x_t are obtained by the following rules derived from Bayes' theorem:

$$p(x_t \mid G_{c,t-1}) = \int_{-\infty}^{\infty} p(x_t \mid x_{t-1}) p(x_{t-1} \mid G_{c,t-1}) dx_{t-1} \tag{16}$$

$$p(x_t \mid G_{c,t}) = p(g_{c,t} \mid x_t) p(x_t \mid G_{c,t-1}) / p(g_{c,t} \mid G_{c,t-1}) \tag{17}$$

$$p(g_{c,t} \mid G_{c,t-1}) = \int_{-\infty}^{\infty} p(g_{c,t} \mid x_t) p(x_t \mid G_{c,t-1}) dx_t \tag{18}$$

The log likelihood function is $ln[p(g_{c,1}, ..., g_{c,T})] = \sum_{t=1}^{T} ln[p(g_{c,t} \mid G_{c,t-1})]$. Maximizing the log likelihood function yields the parameter estimates.

3.3 Estimation Results

In this section, we first report maximum likelihood parameter estimates of confidence measure and consumption growth process. We compare the fit of the Gaussian model (Equations 8-9) with that of the unrestricted DPL model (Equations 12-13). We also consider the fit of three important restricted versions of the DPL model. We then report estimates of the dividend regression (Equations 11 and 15). Lastly, we summarize the estimation results with the alternative method of estimating the unobserved x_t by filtering the dividends data instead of consumption data.

Table 2 reports maximum likelihood estimates for the confidence measure. Our results are comparable to those reported in Bansal and Shaliastovich (2008). The long run mean $\bar{v}$, speed of mean-reversion l and volatility of confidence σ_v are 1.76e-7 (vs 1.44e-7), 0.83 (vs 0.83) and 0.0015 (vs 0.0011), with Bansal and Shalastovich's values in parentheses.[8]

Table 3 reports maximum likelihood estimates for the consumption growth process for " SV Model" in the upper panel and "iid Model" in the lower panel. The fully Gaussian model estimates are reported in the first row. The second row reports results for the unrestricted DPL model. Rows 3-5 report results for three restricted versions of the DPL model as follows. The third row reports estimates for the "symmetric dampening" model, obtained by setting $\beta^c_+ = \beta^c_-$. The fourth row is the "symmetric scale" model, obtained by restricting $\gamma^c_+ = \gamma^c_-$. The fifth row reports estimates for the "symmetric dampening and scale" model, with $\beta^c_+ = \beta^c_-$ and $\gamma^c_+ = \gamma^c_-$.

Briefly, the main findings from the table can be summarized as follows. All statistical inferences are reported at the 0.05 significance level, with some exceptions noted below. The time-invariant mean μ_c is significantly positive for both the Gaussian and DPL models. The coefficient α^c is significantly less than 2 for all the DPL models, which ensures fat-tails for the DPL process. Dampening coefficients β^c_+ and β^c_- are found to be significantly positive. This guarantees finiteness of moments of all orders for the DPL process, thus ensuring finiteness of equilibrium asset prices. The persistence of the long run component ρ is significantly less than 1 for all the models. This is in contrast to the close-to-one value of 0.979 for ρ calibrated by Bansal and Yaron (2004).

An LR test for the Gaussian model versus the unrestricted and restricted DPL models cannot be rejected at the 0.05 significance level using the χ^2 distribution with three degrees of freedom in the category of stochastic volatility (SV) models. However, the Gaussian model is rejected versus the DPL models at the 0.05 significance level in the category of iid models.

We also performed likelihood ratio (LR) tests, with each of the three restricted DPL models in turn as the null model versus the most general unrestricted DPL model given in

[8]Note that Bansal and Shaliastovich (2008) report their values for $\bar{v}$ and σ_v in percentage in their Table 5. $\bar{v}$, l and σ_v in our study correspond to their σ_v^2, ν and σ_w.

Equations (12-15) as the alternative model. In addition, we performed in turn an LR test with the "symmetric dampening and scale" model as the null model versus the "symmetric dampening" and "symmetric scale" models. In every case, we used critical values from the χ^2 distribution with degrees of freedom equal to the number of restrictions needed on the alternative DPL model to obtain the null model under consideration. A 0.05 significance level is used for each of the tests to draw statistical inference. We next discuss each of these hypotheses tests.

(1) **symmetric dampening**

The null hypothesis of "symmetric dampening" tests the restriction $\beta_+^c = \beta_-^c$. With symmetric dampening coefficient β^c, a larger negative jump scale estimate γ_-^c versus a smaller positive jump scale estimate γ_+^c results in negative skewness in the innovations. The estimates are consistent with negative skewness (-0.7576) in the consumption growth data. The LR test statistic for this case is 0.0002 which fails to be rejected.

(2) **symmetric scale**

The null hypothesis of "symmetric scale" tests the restriction $\gamma_+^c = \gamma_-^c$. With symmetric jump scales, a larger positive dampening coefficient β_+^c versus a smaller negative dampening coefficient β_-^c leads to negative skewness of innovations to the consumption growth process. Again, the estimates are consistent with the statistical properties of the consumption growth data. The LR test statistic for this hypothesis is 0.4304 which fails to be rejected.

(3) **symmetric dampening and scale**

The "symmetric dampening and scale" model is obtained by setting $\beta_+^c = \beta_-^c$ and $\gamma_+^c = \gamma_-^c$. The model features symmetric innovations to the consumption growth process. An LR test statistic for this null hypothesis against the unrestricted DPL model is 0.5454 which fails to be rejected. Also, LR test statistics of 0.5452 against the "symmetric dampening" model and 0.1150 against the "symmetric scale" model too cannot be rejected.

In summary, we cannot reject any of the three restricted cases when tested against any less or fully unrestricted DPL models at the 0.05 significance level. In what follows, we choose the "symmetric dampening and scale" model to capture fat tails in the consumption and dividends growth rates process and study its asset pricing implications.

The upper panel of Figure 2 plots the observed and the filtered mean of consumption growth rates (Equation (8)) for the SV Gaussian model. The upper panel of Figure 3 plots similar quantities for the selected SV DPL model. These panels show that both models capture trend consumption growth fairly well.

We report maximum likelihood parameter estimates of dividends growth rate processes given in Equations (11) and (15) in Table 4. Again the upper panel reports the estimates for SV models whereas the lower panel is for iid models. As indicated at the beginning of subsection 3.2, we use the filtered mean of x_t from the Gaussian and DPL models as a proxy for the unobservable persistent component that appears on the right hand sides of these two equations. Results are presented for both the Gaussian model as well as for various versions of the DPL model, as in Table 3 for consumption growth process. For simplicity, we assume a similar DPL structure for innovations to dividends growth as that of innovations to consumption growth. Note that regressions for the alternative models are based on different x_t, filtered from the first step of the estimation procedure. Thus, parameter estimates for the various models exhibit clear differences. Also, a higher likelihood does not necessarily mean a better fit due to the differing x_t for each alternative model.

The lower panel of Figure 2 plots observed dividends and their fitted values in a regression of the former on the filtered mean of the persistent component for the SV Gaussian model. The figure shows that the SV Gaussian model is unable to capture very well fluctuations in dividends growth. The Gaussian model cannot generate significant variability for the years at the end of sample period. The lower panel of Figure 3 plots similar quantities for the selected SV DPL model. The DPL model produces a more reasonable fit to the data, with a somewhat poor fit at the beginning of the sample period.

The estimation results via dividend filtering for the same Gaussian and DPL models are reported for dividend growth process in Table 7 and consumption growth process in Table

8. All conclusions for consumption filtering hold true for dividend filtering, except that persistence of the long run risk ρ is over 0.95 in SV models and over 0.86 in iid models. Thus, the findings for dividend filtering lend more empirical support for the existence of long run risk component than for consumption filtering.

4 Asset Pricing Implications

In this section, we first discuss model parameterization. We then proceed to computing numerically the equilibrium asset prices and returns implied by our model. We examine whether the DPL model exhibits significant improvement over the Gaussian model in order to evaluate the effects of fat tails. We then compare models with stochastic volatility to homoskedastic models to evaluate the effects of stochastic volatility on asset pricing implications. We also report our results under an alternative method for estimating the long run component by filtering dividends data.

4.1 Model Parameterization

Asset pricing formulae summarized in subsection 2.3 show that equilibrium returns and other quantities of interest involve three type of parameters: preference parameters that appear in Equation (1), parameters of the stochastic processes for consumption and dividends growth rates that appear in Equations (12-15), and endogenous (implied) parameters that appear in the approximations to the price-dividend ratios on consumption and market portfolios in Equations (6-7). Stochastic process parameter estimates were reported in subsection 3.3. In this subsection, we elaborate on our choice of preference parameters and our methodology for computing endogenous parameters of price-dividend ratios.

Preference parameters include the risk aversion coefficient γ, the intertemporal elasticity of substitution (IES) ψ, and the time discount factor δ. Our choice of values for these parameters is largely dictated by those used by Bansal and Yaron (2004). However, with dividend filtering, a risk aversion coefficient γ of 4 is sufficient to generate market risk premium comparable that observed in the data. The time discount factor δ is set at 0.998 for decisions made at quarterly intervals. In the next two subsections, we discuss asset pricing implications for various alternative values of γ and ψ.

Sections A1 and A2 in the Appendix discuss how to compute endogenous values for the parameters that appear in the approximations to the gross rates of return to the aggregate consumption and market portfolios appearing in Equations (6-7). These are the average values for the price-dividend ratios $\overline{z}$ and $\overline{z_m}$, and the constants k_0, k_1, k_{0m}, and k_{1m}. Table 5 reports these computations for various alternative values of the preference parameters γ and ψ. The four panels report values for SV Gaussian, SV DPL, iid Gaussian and iid DPL models, respectively. It can be seen that all values for the Gaussian model are largely similar to those for the DPL model.

It is worthwhile to compare our parameter values for the iid Gaussian model to those for the no-fluctuating-uncertainty case in Bansal and Yaron (2004).[9] We briefly summarize the results of comparsion for the case of $\gamma = 10$ and $\psi = 1.5$. Parameter values for our iid Gaussian model with both consumption and dividend filtering are clearly different from those of the equivalent model in Bansal and Yaron (2004). Main reasons are twofold: we use different data and we use the estimated (lower) value for the persistence of the long run risks component instead of setting it to a value of 0.979. The latter study uses twice the value of σ_c and thrice the value of σ_d and ϕ than in our model. Differences in all these values have significant impact on asset pricing implications which we will detail in the following section.

4.2 Effects of Fat Tails

The effects of fat tails are evaluated under iid and stochastic volatility (SV) shocks. With iid shocks, we make a comparison between iid DPL and iid Gaussian models. With SV shocks, we make a comparison between SV DPL and SV Gaussian models.

[9]We thank Dana Kiku for kindly providing these values to us.

Moments of the model-implied rates of return and price-dividend ratio from both Gaussian and DPL models are reported in Table 6. These are the unconditional means and volatilities of the market risk premium and the risk free rate, and the volatility of the price-dividend ratio. These statistics are reported for various values of the risk-aversion coefficient γ and the intertemporal elasticity of substitution (IES) parameter ψ.

First, we assess the effects of fat tails with iid shocks. The third and fourth panels of Table 6 report results for iid Gaussian and iid DPL models, respectively. The expected market risk premium in the iid DPL model is as high as 0.77% for $\gamma = 17.5$ and $\psi = 1.5$, more than twice its corresponding value with iid Gaussian shocks. There are several factors that account for the higher expected market risk premium. The market risk premium is primarily determined by the variances of the innovations to the long run component σ_e^2, the dividend growth σ_d^2 and confidence measure σ_v^2, and the coefficients on these variances. Among their coefficients, ϕ and ρ positively affect the market risk premium. This is evident from the formula for the market risk premium given in Equation A27 in the Appendix. However, estimated values of the first two variances are seen to be similar in magnitude for both models from Table 3-4. The estimated values of ρ for the two models are only marginally different. The higher loading factor on long run risks ϕ in dividends growth contributes to the consistently higher risk premia for the DPL model.

The expected risk free rate and its volatility are somewhat smaller in the iid DPL model compared to the iid Gaussian model. Volatilities of market returns and of the price-dividend ratios in the iid DPL model are more than twice their counterparts in the iid Gaussian model. The inclusion of fat tails in the iid framework does improve empirical performance, especially in generating higher market risk premium and volatility of market returns.

We now proceed to evaluating the effects of fat tails with SV shocks. The first and second panels of Table 6 report results for the SV Gaussian and SV DPL models, respectively. The expected market risk premium in the SV DPL model, reported in the third column of Table 6, is as high as 3.54% compared to zero for the SV Gaussian model. The market risk premium for the SV DPL model shown in Equation A27 in the Appendix is structurally similar to the SV Gaussian model in Equation B12. Larger ϕ, ρ and σ_e positively affect risk premium. Table 3 reports similar values of ρ and σ_e for both models but Table 4 shows more than thrice larger ϕ (2.97) in the SV DPL model than that in the SV Gaussian model (0.78). Hence, the expected market risk premium is vastly higher for the SV DPL model.

The risk free rates and their volatilities in the SV DPL model, reported in the fourth and sixth columns, are of a similar magnitude to the Gaussian model. Equations (B11) and (B13) for the SV Gaussian model and Equations (A24) and (A29) for the SV DPL model share similar structure. The similarity in values is due to the comparable parameter estimates of consumption growth process for both models.

The volatilities of market returns and of the price-dividend ratios in the SV DPL model, reported in the fifth and seventh columns, are significantly higher than in the SV Gaussian model. The better performance of the SV DPL model is attributed to the higher ϕ. This is evident from a comparison between Equations B14 and (A29), and between Equations (B15) and (A17).

Given the results for all four models, we can assess the relative contributions of fat tails and stochastic volatility. We compare the expected market risk premia of the iid Gaussian, iid DPL and SV DPL models, which are of most interest.[10] Note that the expected market risk premium increases from 0.38% for iid Gaussian to 0.77% for iid DPL and to 3.54% for SV DPL model. The first increase is attributed to fat tails, whereas the second increase (2.77%) is due to stochastic volatility. In other words, fat tails (vs. stochastic volatility) contribute to 12.3% (vs. 87.7%) of the increase in total premium.

4.3 Effects of Stochastic Volatility

The effects of stochastic volatility are evaluated with the Gaussian model (without fat tails) and with the DPL model (with fat tails). Under the Gaussian framework, we make a

[10]The SV Gaussian model can barely generate positive risk premium. We do not consider it in assessing the relative contributions of fat tails and stochastic volatility.

comparison between iid Gaussian and SV Gaussian models. Under the DPL framework, we make a comparison between iid DPL and SV DPL models.

The first and third panels of Table 6 report the results for SV Gaussian and iid Gaussian models, respectively. The expected market risk premium in the SV Gaussian model is barely positive for any combination of γ and ψ values considered in the table, whereas the iid Gaussian model can generate 0.38% for $\gamma = 17.5$ and $\psi = 1.5$. The expected risk free rate, volatilities of market returns and of risk free rates and price-dividend ratios are all similar in both SV and iid models. The inclusion of stochastic volatility in the Gaussian framework does not improve its empirical performance.

The second and fourth panels of Table 6 report results for the SV DPL and iid DPL models, respectively. Inclusion of stochastic volatility in the DPL model significantly improves its empirical performance. A detailed comparison between SV DPL and iid DPL models is made in what follows along the third through the seventh columns. We restrict our discussion to the last row of each panel with $\gamma = 17.5$ and $\psi = 1.5$. Similar conclusions hold for other combinations of γ and ψ,

The expected market risk premium in the SV DPL model, reported in the third column of Table 6, is as high as 3.54% versus just 0.77% generated by the iid DPL model. This finding corroborates Bansal and Yaron's (2004) conclusion that fluctuating-uncertainty significantly improves the long run risks model's ability to generate realistic market risk premium. As Equation A27 in the Appendix shows, the market risk premium for the SV DPL model includes a component $(B_{vm} + B_{vf})\bar{v}$ that is absent in the iid DPL model. Given a positive coefficient on unconditional stochastic variance $\bar{v}$, risk premium in the SV DPL model is consequently greater than in the iid DPL model.

The risk free rate in the SV DPL model, reported in the fourth column, shows a similar magnitude to the iid DPL model. A component of stochastic volatility $\bar{v}$ is present in the first two terms B_{0f} and $B_{vf}\bar{v}$ in Equation A24. However, the coefficients of $\bar{v}$ in these two terms are of opposite signs, with the first coefficient being negative and the second positive.

The effects due to stochastic volatility are thus canceling each other in the SV DPL model, thereby making the risk free rate similar to the iid model.

The volatilities of market returns, risk free rates and of price-dividend ratios in the SV DPL model, reported in the fifth to seventh columns, are significantly higher than in the iid DPL model. Especially noteworthy is the fact that volatilities of market returns and of price-dividend ratios in the stochastic volatility model are twice their values in the iid model. The better performance of the SV DPL model can be attributed to the extra component related to the variance of stochastic variance $Var(v_t)$ that does not exist in the iid DPL model. This can be seen from Equations A17, A25 and A29.

4.4 Filtering x_t Using Dividends Data

The discussion so far on the Gaussian and DPL models is based on estimating the long run component x_t through Bayesian filtering using the consumption growth process as the observation equation and the process for x_t as the state transition equation. We now study the robustness of our results to an alternative way of estimating x_t using the dividends growth process, instead of the consumption growth process, as the observation equation.

Maximum likelihood estimation results of the model using dividends process as the observation equation and the x_t process as the state transition equation are reported in Table 7. The table reports results for the Gaussian model and several versions of the DPL model, as in Table 3. Extensive hypotheses testing along the lines reported for that table in subsection 3.3 pin down the "symmetric dampening and scale" DPL model as giving the best fit as for the consumption filtering results. We therefore pursue study of asset pricing implications with this version of the DPL model as the candidate model capturing fat tails.

Maximum likelihood estimation results of the consumption regression equation using x_t obtained by filtering dividends data are reported in Table 8. The table once again reports results for the Gaussian model and several versions of the DPL model.

To illustrate the asset pricing implications with this alternative approach for estimating

x_t, we mainly report results for the parameter combination $\gamma = 4$ and $\psi = 1.5$ for the sake of brevity. The Gaussian model under the alternative approach (see Table 10) can generate 3.98 percent equity risk premium (significantly higher than any value reported in the earlier Gaussian case by filtering consumption growth data for x_t), 2.05 percent risk free rate (compared to 1.84), 18.96 percent volatility of market returns (compared to 2.55), 1.02 percent volatility of risk free rates (compared to 0.69), and 0.26 percent volatility of price-dividend ratios (compared to a value less than 0.01).

The DPL model with dividends filtering also shows similar improvement over the earlier DPL model with consumption filtering based on a comparison of analogous quantities between Table 6 and Table 10. The DPL model also shows similar improvement over the SV Gaussian model based on a comparison between the first and second panels of Table 10. Most significantly, without requiring a high risk aversion, the DPL model under this alternative approach is now able to generate 5.50 percent equity risk premium and 1.25 percent risk free rate, which are close to market data. The main reason for this improvement is that the alternative DPL model now exhibits significantly higher persistence of long-run risks ρ.

In evaluating the relative contribution of fat tails and stochastic volatility to expected market risk premium, we consider the iid Gaussian model, the SV Gaussian model and the SV DPL model. The first two are studied by Bansal and Yaron(2004). Based on the results from Table 10, we find that neither fat tails nor stochastic volatility can explain the total premium. Furthermore, there is an increase of 4.01% per annum in the model-implied premium from iid Gaussian to SV Gaussian, and an increase of 1.54% per annum from SV Gaussian to SV DPL models, respectively. Given that the iid Gaussian model produces close-to-zero premium, fat tails and stochastic volatility roughly contribute 28% and 72% to total premium of 5.50% respectively, assuming simple linear contributions of these two effects.

We now compare our results to those reported in Shaliastovich and Tauchen (2008), Drechsler and Yaron (2008), and Bidarkota and Dupoyet (2007). The first study reports

4.51 percent per annum implied risk premium for their Lévy-process-based model with risk aversion $\gamma = 50$ and IES $\psi = 1.5$. The second paper shows 5.45 percent per annum risk premium for a jump-diffusion model with risk aversion $\gamma = 10$ and IES $\psi = 2$. The last documents 2.72 percent per annum risk premium with risk aversion $\gamma = 7$ assuming the market portfolio pays aggregate consumption as its dividend. Our DPL model with filtering from consumption data cannot generate high enough equity risk premium as reported earlier. However, the premium for the alternative DPL model with filtering from dividends data is computed to be 5.50 percent per annum with significantly lower risk aversion $\gamma = 4$ and $\psi = 1.5$,[11] while all other moments remain comparable to the market data.

Thus, as we have seen above, this alternative approach to estimating x_t produces significantly better empirical results on asset pricing for both the Gaussian and DPL models. The results also reaffirm the earlier conclusion that the DPL model represents a clear improvement over the Gaussian model. However, our mixed results based on univariate filtering (using either consumption or dividends data alone) highlight the need for entertaining bivariate filtering with DPL innovations to consumption and dividends growth rates, which we leave for future research.

5 Conclusions

In this paper, we explore the effects of fat tails on an asset pricing model with long-run risks, stochastic volatility and recursive utility. Following Bansal and Yaron (2004), we model consumption and dividend growth processes with persistent long run components. Given the evidence of leptokurtosis in consumption and dividends data, we introduce non-normality in shocks to their growth rates via a Lévy process, namely the dampened power law (DPL). We derive the asset pricing implications of the resulting model and study the quantitative importance of modeling fat tails empirically.

[11] If we set $\gamma = 4$ and $\psi = 2$ as assumed by Drechsler and Yaron (2008), we obtain 6.14% expected risk premium and 0.56% risk free rate.

When we extract the long run risks component by filtering consumption data, fat tails generate 3.54% expected market risk premium and 1.67% expected risk free rate with the magnitudes of risk aversion and intertemporal elasticity of substitution being 17.5 and 1.5, respectively. By contrast, when we extract the long run risks component by filtering dividends data, the risk premium and risk free rate become 5.50% and 1.25%, both of which are comparable to those observed in the market. Modeling fat tails leads to clear improvement in implied risk premia and volatility of price-dividend ratios, without deterioration in the magnitudes of other moments of interest. The superiority of all four considered models based on dividend filtering results follows the order: SV DPL $>$ SV Gaussian $>$ iid DPL $>$ iid Gaussian in terms of explaining the market risk premium. Clearly, both fat tails and stochastic volatility contribute to a higher risk premium, although neither of them alone can explain the full magnitude of risk premium. The long run risk model with a combination of both features generates the best results.

Extracting the long-run risks component using both consumption and dividends data is more efficient but involves complications arising from consideration of a bivariate DPL and/or filtering process. Also, our asset pricing model assumes that agents not only observe the growth rates of consumption and dividends but also their long run persistent component x_t (although it is assumed in estimation that econometricians do not actually observe the true value of x_t but have to learn about it through a Bayesian filtering process). This may not be entirely realistic (Croce, Lettau, and Ludvigson (2006)). It is worth exploring the effects of fat tails on the long run risks model that treats x_t as unobservable even by agents in the model. Solving the asset pricing model in such an incomplete information setting with fat tails poses a challenge. Bidarkota, Dupoyet, and McCulloch ((2007)) study such a model but without long run risks or recursive utility.

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APPENDIX

A DPL Model Solution

The DPL model is represented by the following set of equations:

$$g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1} \tag{A1}$$

$$x_{t+1} = \rho x_t + \sigma_e \sqrt{v_t} e_{t+1} \tag{A2}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1} \tag{A3}$$

$$v_{t+1} = (1-l)\bar{v} + l v_t + \sigma_v \sqrt{v_t} \epsilon_{t+1} \tag{A4}$$

where $e_t \sim iidN(0, \sigma_e^2)$, $\epsilon_t \sim iidN(0,1)$, $\eta_{c,t} \sim iidDPL(\gamma_+^c, \gamma_-^c, \beta_+^c, \beta_-^c, \alpha^c)$, and $\eta_{d,t} \sim iidDPL(\gamma_+^d, \gamma_-^d, \beta_+^d, \beta_-^d, \alpha^d)$. The DPL processes are defined immediately following Equations (12-15) in the main text.

A.1 Price-Consumption Ratio

The price-consumption and price-dividend ratios z_t and $z_{m,t}$ are the only endogenous variables in the model. Once we solve for these, all other equilibrium quantities of interest can be readily derived. We briefly summarize the procedure for deriving z_t here and $z_{m,t}$ in the next section of the Appendix.

The first-order condition for the representative agent given as Equation (2) in the text can be rewritten for returns on the aggregate consumption portfolio as: $E_t\{exp[\theta ln\delta - \frac{\theta}{\psi} g_{c,t+1} + (\theta-1) r_{a,t+1}]\} = 1$

We substitute for $r_{a,t+1}$ from Equation (6) and $g_{c,t+1}$ from Equation (12) into the above first-order condition to obtain: $E_t\{exp[\theta ln\delta + (\theta - \frac{\theta}{\psi})(\mu_c + x_t + \eta_{c,t+1}) + \theta(k_0 + k_1 z_{t+1} - z_t)]\} = 1$.

We conjecture the following linear solution for the price-consumption ratio as a function of the single state variable x_t and confidence measure v_t in the model: $z_t = b_0 + b_x x_t + b_v v_t$, where b_0, b_x and b_v are constants to be determined.

We substitute this conjectured solution for z_t and the process for the long run component x_t from Equation (A2) into the resulting first-order condition to obtain:

$$\begin{aligned}1 = E_t\{exp[&\theta(ln\delta + (1 - \frac{1}{\psi})\mu_c + k_0 + (k_1 - 1)b_0 + k_1 b_v(1 - l)\bar{v})\\ &+ \theta(1 - \frac{1}{\psi} + k_1 b_x \rho - b_x)x_t + \theta(1 - \frac{1}{\psi})\eta_{c,t+1}\\ &+ [0.5\theta^2 k_1^2 b_x^2 \sigma_e^2 + 0.5\theta^2 k_1^2 b_v^2 \sigma_v^2 + \theta b_v(k_1 l - 1)]v_t]\}\end{aligned}$$

Denote

$$\begin{aligned}A_0 &= \theta(ln\delta + (1 - \frac{1}{\psi})\mu_c + k_0 + (k_1 - 1)b_0 + k_1 b_v(1 - l)\bar{v})\\ A_x &= \theta - \frac{\theta}{\psi} + \theta k_1 b_x \rho - \theta b_x\\ A_\eta &= \theta - \frac{\theta}{\psi}\\ A_v &= 0.5\theta^2 k_1^2 b_x^2 \sigma_e^2 + 0.5\theta^2 k_1^2 b_v^2 \sigma_v^2 + \theta b_v(k_1 l - 1)\end{aligned}$$

We can now rewrite the first-order condition in a simpler way using the above notation as:

$$E_t\{exp[A_0 + A_x x_t + A_\eta \eta_{c,t+1} + A_v v_t]\} = 1 \tag{A5}$$

The conditional expectation term on the lhs of the above equation can be evaluated using the moment generating function (mgf) of innovations following the normal distribution and the more general DPL process considered here. These are given as:

$$E_t[exp(A_\eta \eta_{t+1})] = exp\{\triangle t[\kappa(A_\eta)]\} \tag{A6}$$

$$\kappa(A_\eta) = \Gamma(-\alpha^c)\gamma_+^c[(\beta_+^c - A_\eta)^{\alpha^c} - (\beta_+^c)^{\alpha^c}] + \Gamma(-\alpha^c)\gamma_-^c[(\beta_-^c + A_e)^{\alpha^c} - (\beta_-^c)^{\alpha^c}] + A_e Q \tag{A7}$$

$$Q = \gamma_+^c(\beta_+^c)^{\alpha^c - 1}[\Gamma(-\alpha^c)\alpha^c + \Gamma(1-\alpha^c, \beta_+^c)] - \gamma_-^c(\beta_-^c)^{\alpha^c - 1}[\Gamma(-\alpha^c)\alpha^c + \Gamma(1-\alpha^c, \beta_-^c)] \tag{A8}$$

The cumulant exponent of the DPL innovation $\kappa(A_\eta)$ is derived in Wu (2006a). $\triangle t$ represents agents' decision interval. In our numerical calculations, we set $\triangle t$ to 1 to obtain quarterly results but report annualized values.

We can substitute the above formulae for the mgf's into the simplified first-order condition in Equation A5, and then solve for the constants b_0, b_x and b_v through the method of undetermined coefficients. The solutions for b_0, b_x and b_v are as follows:

$$b_0 = \frac{\theta(ln\delta + k_0 + \mu_c(1 - 1/\psi) + k_1 b_v(1-l)\bar{v}) + \triangle t \kappa(A_\eta)}{\theta(1-k_1)} \tag{A9}$$

$$b_x = \frac{1 - 1/\psi}{1 - k_1\rho} \tag{A10}$$

$$b_v = \frac{0.5\theta k_1^2(b_x^2\sigma_e^2 + b_v^2\sigma_v^2)}{1 - k_1 l} \tag{A11}$$

The approximating constants appearing in Equation (6) in the text k_0 and k_1 are functions of the average level of the price-consumption ratio $\bar{z}$. Evaluating $z_t = b_0 + b_x x_t + b_v v_t$ at $\bar{z}$ and recognizing from Equation (A2) that the average value of x_t is zero yields $\bar{z} = b_0 + b_v\bar{v}$, where $\bar{v}$ is estimated from the data.

Thus, replacing the lhs of Equation A9 with $\bar{z}$ and substituting for b_x from Equation A10 and b_v from Equation A11 into the rhs gives us a (highly) nonlinear equation in $\bar{z}$. We can easily solve this equation numerically for $\bar{z}$. Given $\bar{z}$, k_0 and k_1, and hence b_0, b_x and b_v are readily obtained.

A.2 Price-Dividend Ratio

We briefly summarize the procedure for deriving the price-dividend ratio $z_{m,t}$ on the market portfolio here.

The first-order condition for the representative agent given as Equation (2) in the text can be rewritten for returns on the market portfolio as:

$$E_t[exp(\theta ln\delta - \frac{\theta}{\psi}g_{c,t+1} + (\theta - 1)r_{a,t+1} + r_{m,t+1})] = 1 \quad \text{(A12)}$$

We first substitute the solution to price-consumption ratio $z_t = b_0 + b_x x_t + b_v v_t$ from previous subsection and $g_{c,t+1}$ from Equation 12 into Equation 6 to obtain returns on the aggregate consumption portfolio $r_{a,t+1}$ as follows:

$$r_{a,t+1} = B_{0a} + B_{xa}x_t + B_{va}v_t + B_{ea}\sqrt{v_t}e_{t+1} + B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + \eta_{c,t+1}$$

where

$$B_{0a} = k_0 + (k_1 - 1)b_0 + \mu_c$$
$$B_{xa} = k_1 b_x \rho - b_x + 1 = \frac{1}{\psi}$$
$$B_{va} = (k_1 l - 1)b_v$$
$$B_{ea} = k_1 b_x \sigma_e$$
$$B_{\epsilon a} = k_1 b_v \sigma_v$$

We substitute for $r_{a,t+1}$ from the resulting equation and $r_{m,t+1}$ from Equation 7, $g_{c,t+1}$ from Equation A1 and $g_{d,t+1}$ from Equation A3 into the above first-order condition.

As in deriving the price-consumption ratio, we conjecture the following linear solution for the price-dividend ratio as a function of the state variable x_t and confidence measure v_t in the model: $z_{m,t} = b_{0m} + b_{xm}x_t + b_{vm}v_t$ where b_{0m}, b_{xm} and b_{vm} are constants to be determined.

Substituting this conjectured solution for $z_{m,t}$ into the resulting first-order condition yields:

$$
\begin{aligned}
&E_t\{exp[\theta ln\delta - \frac{\theta}{\psi}g_{c,t+1} + (\theta-1)r_{a,t+1} + r_{m,t+1}]\} = 1 \\
&E_t\{exp[\theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta-1)B_{0a} + k_{0m} + \mu_d + (k_{1m}-1)b_{0m} + k_{1m}b_{vm}\bar{v}(1-l) \\
&\quad + [(\theta-1)B_{xa} - \frac{\theta}{\psi} + \phi + k_{1m}b_{xm}\rho - b_{xm}]x_t + [(\theta-1)B_{ea} + k_{1m}b_{xm}\sigma_e]\sqrt{v_t}e_{t+1} \\
&\quad + [(\theta-1)B_{va} + (k_{1m}l-1)b_{vm}]v_t + [(\theta-1)k_1 + k_{1m}b_{vm}]b_v\sigma_v\sqrt{v_t}\epsilon_{t+1} \\
&\quad + (\theta - 1 - \frac{\theta}{\psi})\eta_{c,t+1} + \eta_{d,t+1}]\} = 1 \qquad \text{(A13)}
\end{aligned}
$$

This can be abbreviated as:

$$
E_t[exp[A_{0m} + A_{xm}x_t + A_{em}\sqrt{v_t}e_{t+1} + A_{\epsilon m}\sqrt{v_t}\epsilon_{t+1} + A_{vm}v_t + A_{\eta m}\eta_{c,t+1} + \eta_{d,t+1}]] = 1
$$

where

$$
\begin{aligned}
A_{0m} &= \theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta-1)B_{0a} + k_{0m} + \mu_d + (k_{1m}-1)b_{0m} + k_{1m}b_{vm}\bar{v}(1-l) \\
A_{xm} &= (\theta-1)B_{xa} - \frac{\theta}{\psi} + \phi + k_{1m}b_{xm}\rho - b_{xm} \\
A_{em} &= (\theta-1)B_{ea} + k_{1m}b_{xm}\sigma_e \\
A_{\epsilon m} &= [(\theta-1)k_1 + k_{1m}b_{vm}]b_v\sigma_v \\
A_{\eta m} &= \theta - 1 - \frac{\theta}{\psi} = -\gamma \\
A_{vm} &= (\theta-1)B_{va} + (k_{1m}l-1)b_{vm}
\end{aligned}
$$

Using the moment generating function for Gaussian random variables, we can rewrite the above equation as:

$$
E_t[exp[A_{0m} + A_{xm}x_t + (0.5A_{em}^2 + 0.5A_{\epsilon m}^2 + A_{vm})v_t + A_{\eta m}\eta_{c,t+1} + \eta_{d,t+1}]] = 1
$$

The constants b_{0m}, b_{xm} and b_{vm} can then be solved by the method of undetermined coefficients:

$$b_{0m} = \frac{\theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta-1)B_{0a} + k_{0m} + \mu_d + k_{1m}b_{vm}\bar{v}(1-l) + \triangle t\kappa_c(A_{\eta m}) + \triangle t\kappa_d(1)}{1-k_{1m}} \tag{A14}$$

$$b_{xm} = \frac{(\theta-1)B_{xa} - \frac{\theta}{\psi} + \phi}{1-k_{1m}\rho} = \frac{\phi - \frac{1}{\psi}}{1-k_{1m}\rho} \tag{A15}$$

$$b_{vm} = \frac{0.5A^2_{em} + 0.5A^2_{\epsilon m} + (\theta-1)B_{va}}{1-k_{1m}l} \tag{A16}$$

where $\kappa_c(A_{\eta m})$, $\kappa_d(1)$ are the cumulant exponents of the DPL innovation to consumption and dividend growth respectively. $\kappa_d(1)$ is computed through Equation A7, but with the DPL parameters to the innovation to consumption growth $\gamma^c_+, \gamma^c_-, \beta^c_+, \beta^c_-, \alpha^c$ being replaced by the parameters to dividend growth $\gamma^d_+, \gamma^d_-, \beta^d_+, \beta^d_-, \alpha^d$.

The approximating constants appearing in Equation 7 in the text k_{0m} and k_{1m} are functions of the average level of the price-dividend ratio $\bar{z_m}$. Evaluating $z_{m,t} = b_{0m} + b_{xm}x_t + b_{vm}v_t$ at $\bar{z_m}$ and recognizing from Equation 13 that the average value of x_t is zero yields $\bar{z_m} = b_{0m} + b_{vm}\bar{v}$, where $\bar{v}$ is estimated from the data.

Thus, replacing the lhs of Equation A14 with $\bar{z_m}$ and substituting for b_{xm} from Equation A15 into the rhs give us a nonlinear equation in $\bar{z_m}$. We can solve this equation numerically for $\bar{z_m}$. Given $\bar{z_m}$, k_{0m} and k_{1m}, and hence b_{0m}, b_{xm} and b_{vm} are readily obtained.

Given $z_{m,t} = b_{0m} + b_{xm}x_t + b_{vm}v_t$, variance of price-dividend ratio $z_{m,t}$ can be easily obtained as

$$Var(z_{m,t}) = b^2_{xm}Var(x_t) + b^2_{vm}Var(v_t) \tag{A17}$$

where $Var(x_t) = \frac{\sigma^2_e\bar{v}}{1-\rho^2}$ and $Var(v_t) = \frac{\sigma^2_v\bar{v}}{1-l^2}$ are obtained by taking variance operation on both sides of $x_{t+1} = \rho x_t + \sigma_e\sqrt{v_t}e_{t+1}$ and $v_{t+1} = (1-l)\bar{v} + lv_t + \sigma_v\sqrt{v_t}\epsilon_{t+1}$.

A.3 Returns on Aggregate Consumption Portfolio

Returns on the aggregate consumption portfolio $r_{a,t+1}$ are given in Equation (6). Using $z_t = b_0 + b_x x_t + b_v v_t$ and the DPL process for $g_{c,t+1}$ from Equation (A1) yields:

$$r_{a,t+1} = B_{0a} + B_{xa}x_t + B_{va}v_t + B_{ea}\sqrt{v_t}e_{t+1} + B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + \eta_{c,t+1} \tag{A18}$$

where $B_{0a} = k_0 + (k_1 - 1)b_0 + \mu_c$, $B_{xa} = k_1 b_x \rho - b_x + 1 = \frac{1}{\psi}$, $B_{va} = (k_1 l - 1)b_v$, $B_{ea} = k_1 b_x \sigma_e$ and $B_{\epsilon a} = k_1 b_v \sigma_v$.

Innovations to returns on the aggregate consumption portfolio $r_{a,t+1} - E[r_{a,t+1}]$ can be expressed as:

$$\begin{aligned} r_{a,t+1} - E[r_{a,t+1}] &= B_{xa}(x_t - E[x_t]) + B_{va}(v_t - E[v_t]) + B_{ea}(\sqrt{v_t}e_{t+1} - E[\sqrt{v_t}e_{t+1}]) \\ &+ B_{\epsilon a}(\sqrt{v_t}\epsilon_{t+1} - E[\sqrt{v_t}\epsilon_{t+1}]) + \eta_{c,t+1} - E[\eta_{c,1+1}] \\ &= B_{xa}x_t + B_{va}(v_t - \bar{v}) + B_{ea}\sqrt{v_t}e_{t+1} + B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + \eta_{c,t+1} \end{aligned} \tag{A19}$$

by recognizing that $E[x_t]$, $E[e_{t+1}]$, $E[\epsilon_{t+1}]$, $E[\eta_{c,t+1}]$ are equal to zero and $E[v_t] = \bar{v}$.

A.4 Pricing Kernel (IMRS)

The (logarithm of the) pricing kernel m_{t+1} is given in Equation (5) in the main text. Substituting for the DPL consumption process from Equation (A1) and $r_{a,t+1}$ from Equation (A18) derived in the previous section of this Appendix into the formula for the pricing kernel yields:

$$
\begin{aligned}
m_{t+1} &= \theta ln\delta - \frac{\theta}{\psi}g_{c,t+1} + (\theta-1)r_{a,t+1} \\
&= \theta ln\delta - \frac{\theta}{\psi}(\mu_c + x_t + \eta_{c,t+1}) + (\theta-1)(B_{0a} + B_{xa}x_t + B_{va}v_t \\
&+ B_{ea}\sqrt{v_t}e_{t+1} + B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + \eta_{c,t+1}) \\
&= \theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta-1)B_{0a} + ((\theta-1)B_{xa} - \frac{\theta}{\psi})x_t + (\theta-1)B_{va}v_t \\
&+ (\theta-1)B_{ea}\sqrt{v_t}e_{t+1} + (\theta-1)B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + (\theta - 1 - \frac{\theta}{\psi})\eta_{c,t+1} \qquad \text{(A20)}
\end{aligned}
$$

Innovations to the pricing kernel are given as:

$$
m_{t+1} - E_t(m_{t+1}) = (\theta-1)B_{ea}\sqrt{v_t}e_{t+1} + (\theta-1)B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + (\theta - 1 - \frac{\theta}{\psi})\eta_{c,t+1} \qquad \text{(A21)}
$$

The conditional and unconditional variances of the pricing kernel can then be obtained as:

$$
\begin{aligned}
Var_t(m_{t+1}) &= E_t[(m_{t+1} - E_t(m_{t+1}))^2] = (\theta-1)^2(B_{ea}^2 + B_{\epsilon a}^2)v_t + (\theta - 1 - \frac{\theta}{\psi})^2\sigma_\eta^2 \\
Var(m_{t+1}) &= ((\theta-1)B_{xa} - \frac{\theta}{\psi})^2 Var(x_t) + (\theta-1)^2 B_{va}^2 Var(v_t) \\
&+ (\theta-1)^2(B_{ea}^2 + B_{\epsilon a}^2)\bar{v} + (\theta - 1 - \frac{\theta}{\psi})^2 Var(\eta_c) \qquad \text{(A22)}
\end{aligned}
$$

where $Var(x_t) = \frac{\sigma_e^2\bar{v}}{1-\rho^2}$, $Var(v_t) = \frac{\sigma_v^2\bar{v}}{1-l^2}$, and $Var(\eta_c) = \triangle t\Gamma(2-\alpha^c)[\gamma_+^c(\beta_+^c)^{\alpha^c-2} + \gamma_-^c(\beta_-^c)^{\alpha^c-2}]$ is the second moment of the consumption growth innovation following the DPL process.

A.5 Risk Free Rate

The risk free asset pays one unit of consumption good as dividends every period. The first-order condition for the representative agent given as Equation (2) in the text can be rewritten for risk free returns as: $E_t[exp(m_{t+1}r_{f,t+1})] = 1$. Recognizing that the risk free rate $r_{f,t+1}$ is known as of time t, and using Equation (A20) for m_{t+1} and Equation (A18) for $r_{a,t+1}$, we can derive the risk free rate as:

$$
\begin{aligned}
exp(r_{f,t+1}) &= 1/E_t\{exp[\theta ln\delta + (\theta-1)B_{0a} - \frac{\theta}{\psi}\mu_c + [(\theta-1)B_{xa} - \frac{\theta}{\psi}]x_t \\
&\quad + (\theta-1)B_{ea}\sqrt{v_t}e_{t+1} + (\theta-1)B_{\epsilon a}\sqrt{v_t}\epsilon_{t+1} + (\theta-1)B_{va}v_t + (\theta-1-\frac{\theta}{\psi})\eta_{c,t+1})\} \\
r_{f,t+1} &= -B_{0f} - B_{xf}x_t - B_{vf}v_t - \triangle t\kappa(B_{\eta f}) \qquad \text{(A23)}
\end{aligned}
$$

where

$$
\begin{aligned}
B_{0f} &= \theta ln\delta + (\theta-1)B_{0a} - \frac{\theta}{\psi}\mu_c \\
B_{xf} &= (\theta-1)B_{xa} - \frac{\theta}{\psi} \\
B_{vf} &= 0.5(\theta-1)^2(B_{ea}^2 + B_{\epsilon a}^2) + (\theta-1)B_{va} \\
B_{\eta f} &= \theta - 1 - \frac{\theta}{\psi} = -\gamma
\end{aligned}
$$

The second equality for B_{0f} is obtained by substituting Equation (A9) for b_0 into the rhs of the first equation.

Using $E[x_t] = 0$ and $A_\eta = \theta - \frac{\theta}{\psi} = 1 - \gamma$, the unconditional expectation of the risk free rate $E[r_{f,t}]$ is given by:

$$E[r_{f,t+1}] = -B_{0f} - B_{vf}\bar{v} - \triangle t\kappa(B_{\eta f}) \qquad \text{(A24)}$$

Unconditional variance of the risk free rate $Var(r_{f,t+1})$ can be easily obtained from Equation (A23) as:

$$Var(r_{f,t+1}) = B_{xf}^2 Var(x_t) + B_{vf}^2 Var(v_t) = \frac{B_{xf}^2}{1-\rho^2}\sigma_e^2 \qquad \text{(A25)}$$

where $Var(x_t) = \frac{1}{1-\rho^2}\sigma_e^2$ and $Var(v_t) = \frac{\sigma_v^2\bar{v}}{1-l^2}$ as shown in Section A.2.

A.6 Market Risk Premium

Returns on the market portfolio $r_{m,t+1}$ are given in Equation (7). Using $z_{m,t} = b_{0m} + b_{xm}x_t + b_{xv}v_t$ and substituting Equations (A1) and (A3) for $g_{c,t+1}$ and $g_{d,t+1}$ respectively yields:

$$\begin{aligned} r_{m,t+1} &= k_{0m} + (k_{1m} - 1)b_{0m} + \mu_d + k_{1m}b_{vm}\bar{v}(1 - l) + (k_{1m}b_{xm}\rho - b_{xm} + \phi)x_t \\ &\quad + k_{1m}b_{xm}\sigma_e\sqrt{v_t}e_{t+1} + k_{1m}b_{vm}\sigma_v\sqrt{v_t}\epsilon_{t+1} + (k_{1m}l - 1)b_{vm}v_t + \eta_{d,t+1} \\ &= B_{0m} + B_{xm}x_t + B_{em}\sqrt{v_t}e_{t+1} + B_{\epsilon m}\sqrt{v_t}\epsilon_{t+1} + B_{vm}v_t + \eta_{c,t+1} \end{aligned} \tag{A26}$$

where

$$\begin{aligned} B_{0m} &= k_{0m} + (k_{1m} - 1)b_{0m} + \mu_d + k_{1m}b_{vm}\bar{v}(1 - l) \\ B_{xm} &= k_{1m}b_{xm}\rho - b_{xm} + \phi \\ B_{em} &= k_{1m}b_{xm}\sigma_e \\ B_{\epsilon m} &= k_{1m}b_{\epsilon m}\sigma_v \\ B_{vm} &= (k_{1m}l - 1)b_{vm} \end{aligned}$$

Subtracting the expected risk free rate in Equation (A24) from returns on the market portfolio in Equation A26 yields conditional and unconditional market risk premium:

$$\begin{aligned} E_t[r_{m,t+1} - r_{f,t+1}] &= B_{0m} + B_{0f} + (B_{vm} + B_{vf})v_t + (B_{xm} + B_{xf})x_t \\ E[r_{m,t+1} - r_{f,t+1}] &= B_{0m} + B_{0f} + (B_{vm} + B_{vf})\bar{v} + \triangle t\kappa_c(B_{\eta f}) \end{aligned} \tag{A27}$$

A.7 Variance of Market Returns

We can derive the innovations to market returns as: $r_{m,t+1} - E_t[r_{m,t+1}] = k_{1m}b_{xm}\sigma_e e_{t+1} + \eta_{d,t+1}$.

The conditional and unconditional variances of market returns can then be obtained as follows:

$$\begin{aligned}Var_t(r_{m,t+1}) &= E_t[r_{m,t+1} - E_t[r_{m,t+1}]]^2 \\ &= (B_{em}^2 + B_{\epsilon m}^2)v_t + Var(\eta_d) \qquad \text{(A28)} \\ Var(r_{m,t+1}) &= B_{xm}^2 Var(x_t) + (B_{em}^2 + B_{\epsilon m}^2)\bar{v} + B_{vm}^2 Var(v_t) + Var(\eta_d) \qquad \text{(A29)}\end{aligned}$$

where $Var(x_t) = \frac{\sigma_e^2 \bar{v}}{1-\rho^2}$, $Var(v_t) = \frac{\sigma_v^2 \bar{v}}{1-l^2}$ and $Var(\eta_d) = \triangle t \Gamma(2-\alpha^d)[\gamma_+^d(\beta_+^d)^{\alpha^d-2} + \gamma_-^d(\beta_-^d)^{\alpha^d-2}]$.

B Gaussian Model Solution

The Gaussian model with long-run risks and stochastic volatility is represented by the following set of equations:

$$g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1} \qquad \text{(B1)}$$

$$x_{t+1} = \rho x_t + \sigma_e \sqrt{v_t} e_{t+1} \qquad \text{(B2)}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1} \qquad \text{(B3)}$$

$$v_{t+1} = (1-l)\bar{v} + l v_t + \sigma_v \sqrt{v_t} \epsilon_{t+1} \qquad \text{(B4)}$$

where $e_t \sim iidN(0,1)$, $\epsilon_t \sim iidN(0,1)$, $\eta_{c,t} \sim iidN(0,\sigma_c^2)$, and $\eta_{d,t} \sim iidN(0,\sigma_d^2)$. A special case is the iid Gaussian model, in which v_t is trivially constant or equivalently $l, \epsilon_t = 0$. The solution follows exactly the same method as that for the DPL model. Without repeating the procedure, we can obtain the results for the Gaussian model by replacing $\kappa_c(s)$, $\kappa_d(s)$, $Var(\eta_c)$, $Var(\eta_d)$ with $0.5s^2\sigma_c^2$, $0.5s^2\sigma_c^2$, σ_c^2 and σ_d^2, respectively.

We state below formulae for equilibrium quantities of interest for the Gaussian model: namely price-consumption ratio z_t, price-dividend ratio $z_{m,t}$, expected risk-free rate $E[r_{f,t+1}]$, expected market risk premium $E[r_{m,t+1} - r_{f,t+1}]$, variances of risk free rate $Var(r_{f,t+1})$, market return $Var(r_{m,t+1})$, and price-dividend ratio $Var(z_{m,t})$.

Price-consumption ratio $z_t = b_0 + b_x x_t + b_v v_t$ with

$$b_0 = \frac{\theta(ln\delta + k_0 + \mu_c(1 - 1/\psi) + k_1 b_v(1 - l)\bar{v}) + 0.5(\theta(1 - k_1))^2\sigma_c^2}{\theta(1 - k_1)} \tag{B5}$$

$$b_x = \frac{1 - 1/\psi}{1 - k_1\rho} \tag{B6}$$

$$b_v = \frac{0.5\theta k_1^2(b_x^2\sigma_e^2 + b_v^2\sigma_v^2)}{1 - k_1 l} \tag{B7}$$

Price-dividend ratio $z_{m,t} = b_{0m} + b_{xm}x_t + b_{vm}v_t$ with

$$b_{0m} = \frac{\theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta - 1)B_{0a} + k_{0m} + \mu_d + k_{1m}b_{vm}\bar{v}(1 - l) + 0.5A_{\eta m}^2\sigma_c^2 + 0.5\sigma_d^2}{1 - k_{1m}} \tag{B8}$$

$$b_{xm} = \frac{(\theta - 1)B_{xa} - \frac{\theta}{\psi} + \phi}{1 - k_{1m}\rho} = \frac{\phi - \frac{1}{\psi}}{1 - k_{1m}\rho} \tag{B9}$$

$$b_{vm} = \frac{0.5A_{em}^2 + 0.5A_{\epsilon m}^2 + (\theta - 1)B_{va}}{1 - k_{1m}l} \tag{B10}$$

Expected risk-free rate, market risk premium and variances:

$$E[r_{f,t+1}] = -B_{0f} - B_{vf}\bar{v} - 0.5B_{\eta f}^2\sigma_c^2 \tag{B11}$$

$$E[r_{m,t+1}] - r_{f,t+1} = B_{0m} + B_{0f} + (B_{vm} + B_{vf})\bar{v} + 0.5B_{\eta f}^2\sigma_c^2 \tag{B12}$$

$$Var(r_{f,t+1}) = B_{xf}^2 Var(x_t) + B_{vf}^2 Var(v_t) \tag{B13}$$

$$Var(r_{m,t+1}) = B_{xm}^2 Var(x_t) + (B_{em}^2 + B_{\epsilon m}^2)\bar{v} + B_{vm}^2 Var(v_t) + \sigma_d^2 \tag{B14}$$

$$Var(z_{m,t}) = b_{xm}^2 Var(x_t) + b_{vm}^2 Var(v_t) \tag{B15}$$

where $Var(x_t) = \frac{\sigma_e^2\bar{v}}{1-\rho^2}$ and $Var(v_t) = \frac{\sigma_v^2\bar{v}}{1-l^2}$.

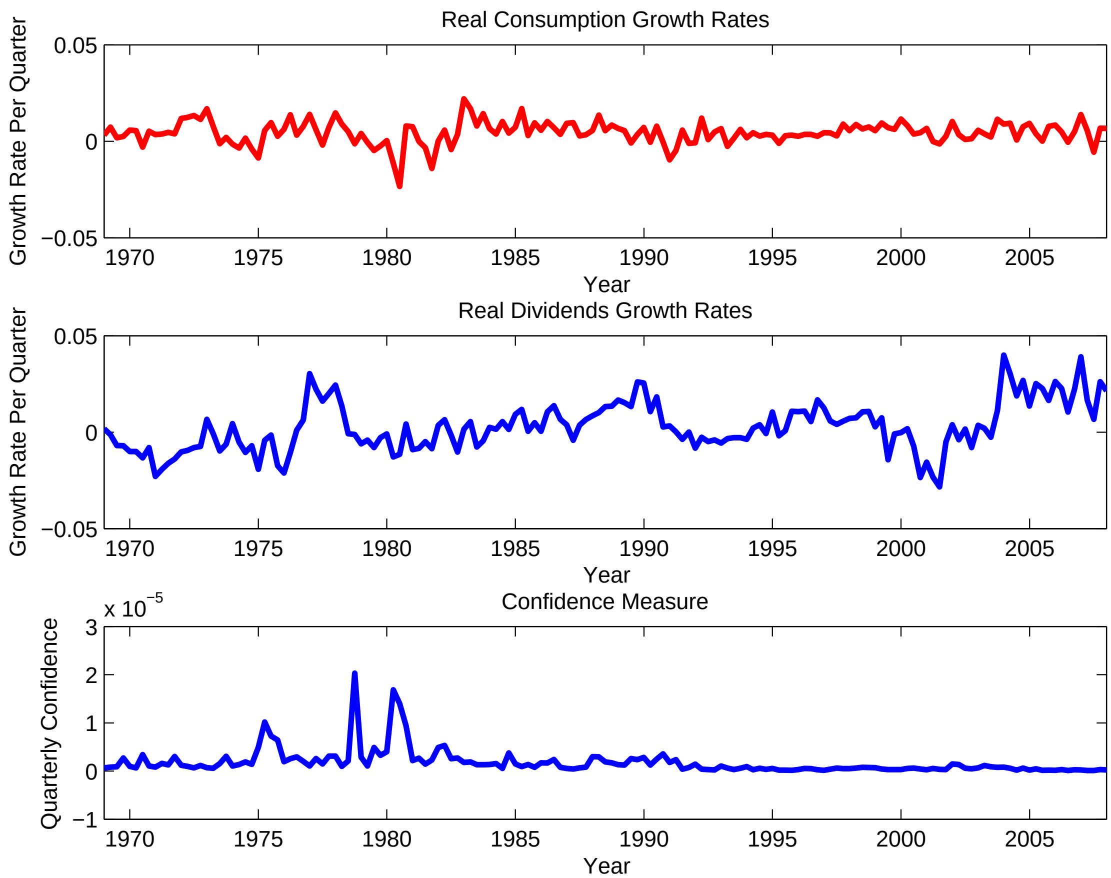

Figure 1: Quarterly consumption and dividends growth rates

Quarterly consumption and dividends growth rates span the period 1968:IV through 2007:IV. Consumption includes non-durable goods and services from the NIPA tables. Dividends, paid toward the S&P 500 index, are obtained from Robert Shiller's website. Nominal consumption and dividends are deflated by the CPI series to obtain real quantities.

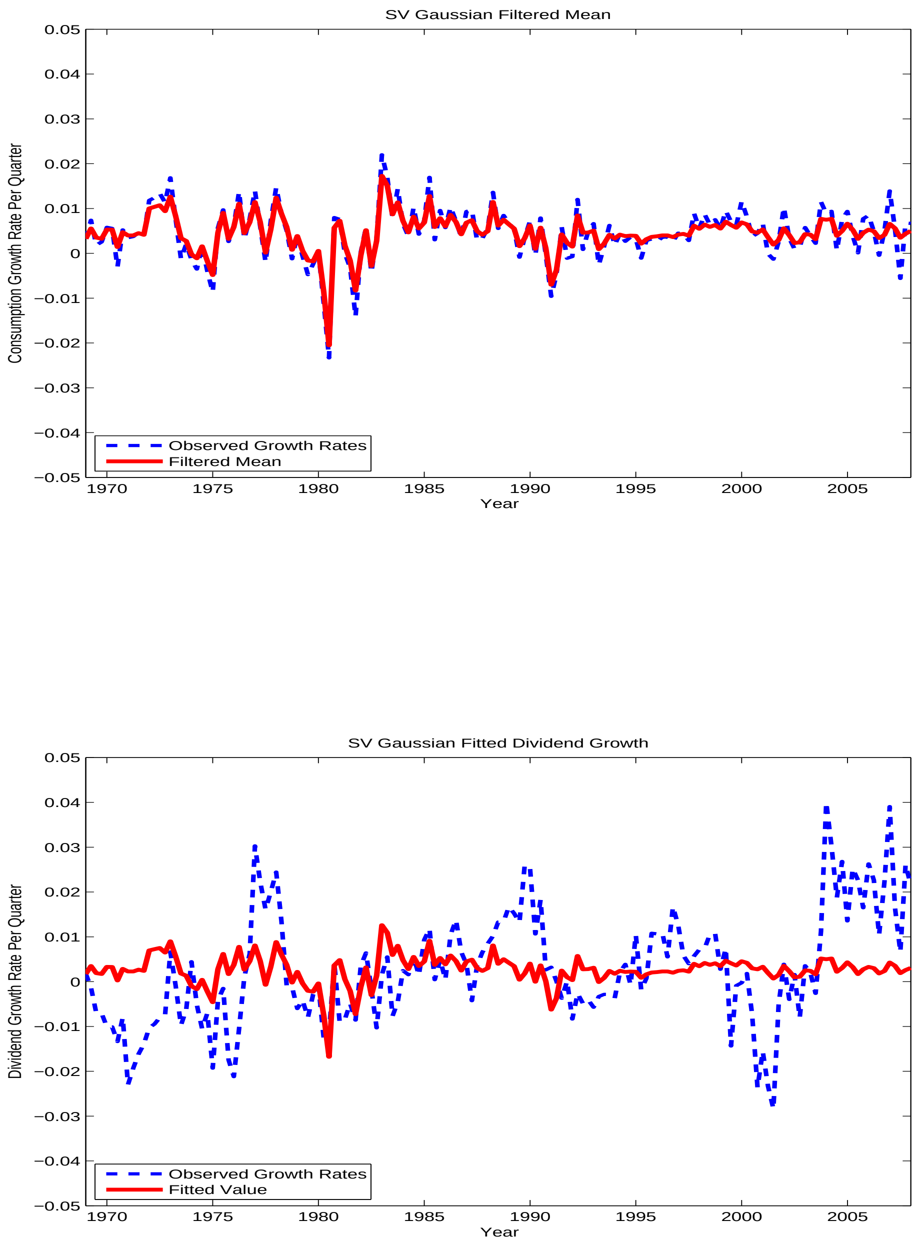

Figure 2: Filtered consumption and fitted dividend growth: the SV Gaussian model
The upper panel plots observed consumption and its filtered mean for the SV Gaussian model, where innovations to consumption growth rates are assumed to be Gaussian and innovations to the long-run risk component exhibit stochastic volatility. The lower panel plots observed dividends and their fitted values in a regression of the former on the filtered mean of the persistent component (this is the filtered mean series plotted in the upper panel, adjusted for a non-zero time-invariant mean as in Equation (8)).

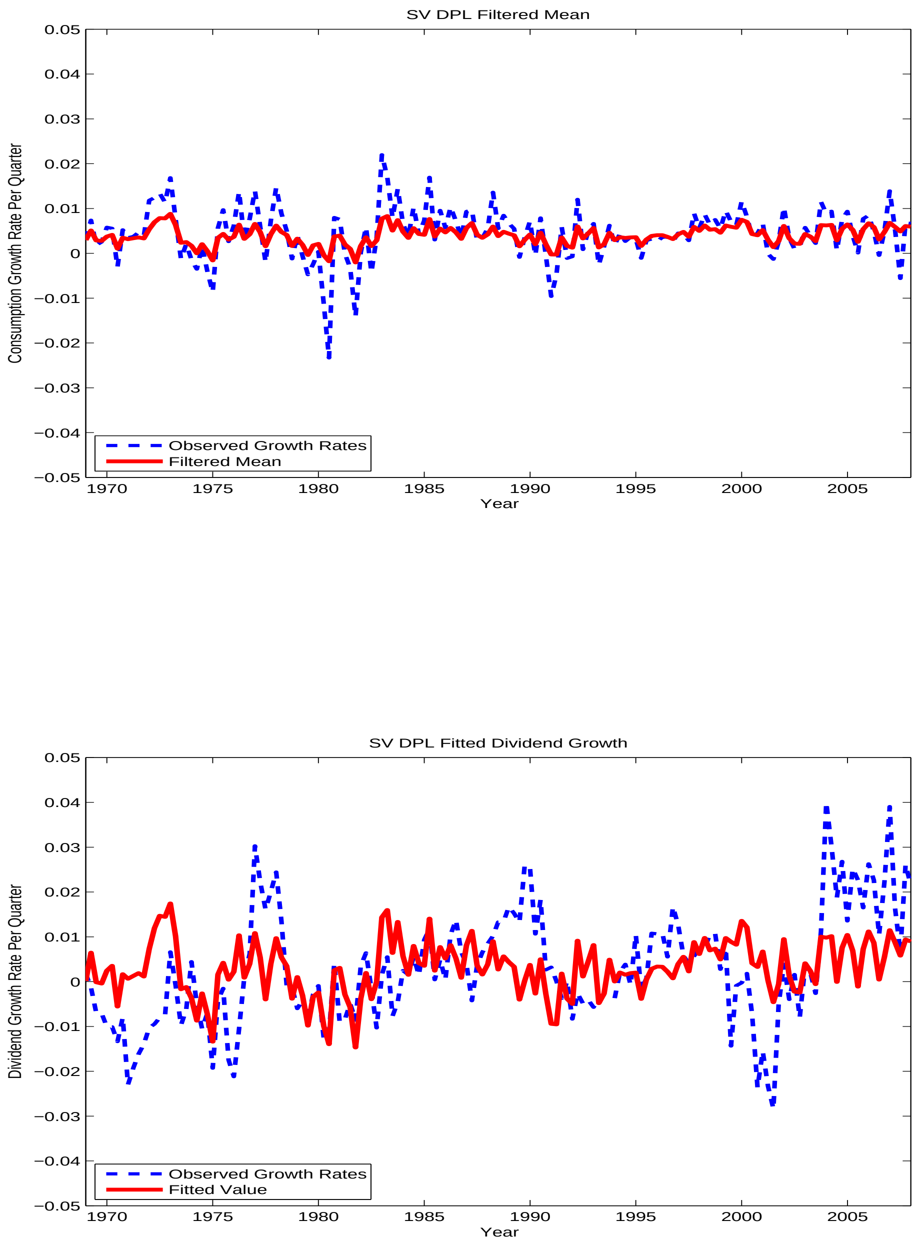

Figure 3: Filtered consumption and fitted dividend growth: the SV DPL model
The upper panel plots observed consumption and its filtered mean for the SV DPL model, where iid innovations to consumption growth rates are assumed to follow the dampened power law and innovations to the long-run risk component exhibit stochastic volatility. The lower panel plots observed dividends and their fitted values in a regression of the former on the filtered mean of the persistent component (this is the filtered mean series plotted in the upper panel, adjusted for a non-zero time-invariant mean as in Equation (12)).

Table 1: Summary Statistics for Consumption and Dividends

	Mean	Std. Dev.	Skewness	Kurtosis	J-B Test
Consumption Growth Rates	0.0045	0.0059	-0.7576	6.3188	87.0691
	0.0005	0.0003	0.0000	0.0000	0.0000
Dividends Growth Rates	0.0027	0.0126	0.3620	3.2131	3.7263
	0.0010	0.0007	0.0320	0.2928	0.1552

The table presents summary statistics for quarterly real per capita consumption and dividends growth rates over the period 1968:IV-2007:IV. Numbers in the second row of each panel are standard errors for columns 1 and 2, and p-values for columns 3-5. Consumption data, for non-durable goods and services, are obtained from NIPA tables. Dividends, paid toward the S&P 500 index, are obtained from Robert Shiller's website.

Table 2: Parameter Estimates for Confidence Measure

$\bar{v}$	l	σ_v
1.76E-06	0.8332	0.001470
6.81E-07	3.37E-06	8.73E-05

[1] The confidence measure v_t follows:
$v_{t+1} = (1-l)\bar{v} + lv_t + \sigma_v\sqrt{v_t}\epsilon_{t+1}$.

[2] The table presents parameter estimates for quarterly confidence over the period 1968:IV-2007:IV. Numbers in the second row are standard errors.

Table 3: Maximum Likelihood Parameter Estimates of Consumption Growth Process with Consumption Filtering

SV Model	μ_c	σ_e	ρ	σ_c							LogL
Gaussian	0.0045	3.2465	0.5378	0.0033							605.3985
	0.0006	0.4648	0.1138	0.0004							
DPL	μ_c	σ_e	ρ	γ^c_+	γ^c_-	$\gamma^c_+ = \gamma^c_-$	β^c_+	β^c_-	$\beta^c_+ = \beta^c_-$	α^c	LogL
Unrestricted	0.0047	3.5981	0.4863	0.0000	0.0001		68.9295	68.9296		1.5997	605.9676
DPL	0.0005	0.0000	0.0871	0.0001	0.0001		0.0124	0.1051		0.2089	
Sym. Damp.	0.0048	3.6051	0.4767			0.0001			65.7091	1.5239	605.6949
and Scale	0.0004	0.0000	0.0001			0.0000			0.0138	0.0008	
Sym. Damp.	0.0047	3.5981	0.4863	0.0000	0.0001				68.9275	1.5997	605.9675
	0.0010	0.0000	0.0001	0.0000	0.0000				0.0001	0.0000	
Sym. Scale	0.0047	3.6064	0.4841			0.0000	65.5833	50.5554		1.6121	605.7524
	0.0003	0.0000	0.1088			0.0001	0.3302	0.1328		0.2089	
IID Model	μ_c	σ_e	ρ	σ_c							LogL
Gaussian	0.0045	0.0027	0.7230	0.0044							590.4368
	0.0009	0.0011	0.1643	0.0007							
DPL	μ_c	σ_e	ρ	γ^c_+	γ^c_-	$\gamma^c_+ = \gamma^c_-$	β^c_+	β^c_-	$\beta^c_+ = \beta^c_-$	α^c	LogL
Unrestricted	0.0045	0.0026	0.6526	0.0006	0.0005		101.0059	37.9989		1.3293	597.1453
DPL	0.0006	0.0000	0.0001	0.0002	0.0000		0.0000	0.0000		0.0000	
Sym. Damp.	0.0047	0.0027	0.6451			0.0007			67.6433	1.2752	596.7125
and Scale	0.0007	0.0000	0.0988			0.0006			0.6274	0.1439	
Sym. Damp.	0.0046	0.0027	0.6411	0.0005	0.0007				67.6161	1.3080	596.8133
	0.0038	0.0000	0.5665	0.0011	0.0019				0.3846	0.4014	
Sym. Scale	0.0045	0.0028	0.6208			0.0004	101.5085	38.5563		1.3569	597.0889
	0.0010	0.0000	0.1002			0.0006	0.0050	0.2240		0.2240	

1 Consumption Growth Process: $g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1}$. DPL model: $\eta_{c,t+1} \sim iidDPL(\gamma^c_+, \gamma^c_-, \beta^c_+, \beta^c_-, \alpha^c)$. Gaussian model: $\eta_{c,t+1} \sim iidN(0, \sigma_c^2)$ with state transition: $x_{t+1} = \rho x_t + \sigma_e \sqrt{v_t} e_{t+1}$. $e_{t+1} \sim iidN(0, 1)$.

2 "Sym. Damp." and "Sym. Scale" refer to the DPL models with "Symmetric Dampening" and "Symmetric Scale". Parameter estimates are reported in the first row, and standard errors in the second.

Table 4: Maximum Likelihood Parameter Estimates of Dividend Growth Process with Consumption Filtering

SV Model	μ_d	ϕ	σ_d							LogL
Gaussian	0.0027	0.7770	0.0121							470.4655
	0.0010	0.2273	0.0007							
DPL	μ_d	ϕ	γ_+^d	γ_-^d	$\gamma_+^d = \gamma_-^d$	β_+^d	β_-^d	$\beta_+^d = \beta_-^d$	α^d	LogL
Unrestricted	0.0043	3.0938	0.0000	0.0000		99.9893	106.9107		1.9833	474.3357
DPL	0.0000	0.0000	0.0002	0.0004		0.0194	37.0325		0.4693	
Sym. Damp.	0.0055	2.9709			0.0130			1377.4860	1.4101	475.0847
and Scale	0.0000	0.0000			0.0002			0.0012	0.0094	
Sym. Damp.	0.0038	2.1608	0.0001	0.0000				43.7430	1.8866	476.1616
	0.0000	0.0002	0.0002	0.0002				0.9497	0.0840	
Sym. Scale	0.0040	2.5698			0.0023	568.9246	401.9506		1.5463	475.6820
	0.0000	0.0001			0.0004	5.1005	3.8646		0.0070	
IID Model	μ_d	ϕ	σ_d							LogL
Gaussian	0.0027	1.3259	0.0119							472.7206
	0.0010	0.3254	0.0007							
DPL	μ_d	ϕ	γ_+^d	γ_-^d	$\gamma_+^d = \gamma_-^d$	β_+^d	β_-^d	$\beta_+^d = \beta_-^d$	α^d	LogL
Unrestricted	0.0041	2.8912	0.0002	0.0079		249.7468	313.0983		1.4597	467.7576
DPL	0.0000	0.0014	0.0071	0.0055		5.6271	3.1875		0.0434	
Sym. Damp.	0.0036	3.2371			0.0000			81.3665	1.9723	464.8407
and Scale	0.0000	0.0015			0.0000			0.0477	0.0722	
Sym. Damp.	0.0047	3.3556	0.0013	0.0198				137.5070	1.2501	462.1866
	0.0000	0.0000	0.0001	0.0000				0.0049	0.0197	
Sym. Scale	0.0031	3.0126			0.0000	65.7515	83.6359		1.9968	465.9368
	0.0000	0.0000			0.0000	1.9720	2.4559		0.0043	

1 Dividend Growth Process: $g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1}$. DPL model: $\eta_{d,t+1} \sim iidDPL(\gamma_+^d, \gamma_-^d, \beta_+^d, \beta_-^d, \alpha^d)$. Gaussian model: $\eta_{d,t+1} \sim iidN(0, \sigma_d^2)$

2 "Sym. Damp." and "Sym. Scale" refer to the DPL models with "Symmetric Dampening" and "Symmetric Scale". Parameter estimates are reported in the first row, and standard errors in the second.

Table 5: Parameters Determining Price-Consumption and Price-Dividend Ratios

γ	ψ	$\bar{z}$	k_0	k_1	$\bar{z_m}$	k_{0m}	k_{1m}
SV Gaussian							
7.5	0.5	5.083	0.038	0.994	4.863	0.045	0.992
7.5	1.5	7.399	0.005	0.999	6.154	0.015	0.998
10	0.5	5.106	0.037	0.994	4.894	0.044	0.993
10	1.5	7.322	0.005	0.999	6.179	0.015	0.998
15	0.5	5.171	0.035	0.994	4.993	0.040	0.993
15	1.5	7.136	0.006	0.999	6.310	0.013	0.998
17.5	0.5	NA	NA	NA	NA	NA	NA
17.5	1.5	NA	NA	NA	NA	NA	NA
SV DPL							
7.5	0.5	5.054	0.038	0.994	5.108	0.037	0.994
7.5	1.5	7.502	0.005	0.999	8.311	0.002	1.000
10	0.5	5.082	0.038	0.994	5.068	0.038	0.994
10	1.5	7.401	0.005	0.999	7.197	0.006	0.999
15	0.5	5.154	0.035	0.994	4.902	0.044	0.993
15	1.5	7.179	0.006	0.999	5.929	0.018	0.997
17.5	0.5	5.224	0.033	0.995	4.625	0.055	0.990
17.5	1.5	6.967	0.007	0.999	4.882	0.044	0.992
IID Gaussian							
7.5	0.5	5.089	0.037	0.994	4.833	0.046	0.992
7.5	1.5	7.377	0.005	0.999	6.026	0.017	0.998
10	0.5	5.112	0.037	0.994	4.848	0.046	0.992
10	1.5	7.304	0.006	0.999	5.995	0.017	0.998
15	0.5	5.159	0.035	0.994	4.877	0.044	0.992
15	1.5	7.172	0.006	0.999	5.937	0.018	0.997
17.5	0.5	5.184	0.034	0.994	4.892	0.044	0.993
17.5	1.5	7.112	0.007	0.999	5.909	0.019	0.997
IID DPL							
7.5	0.5	5.077	0.038	0.994	4.955	0.042	0.993
7.5	1.5	7.422	0.005	0.999	6.555	0.011	0.999
10	0.5	5.106	0.037	0.994	4.968	0.041	0.993
10	1.5	7.327	0.005	0.999	6.453	0.012	0.998
15	0.5	5.167	0.035	0.994	4.996	0.040	0.993
15	1.5	7.159	0.006	0.999	6.279	0.014	0.998
17.5	0.5	5.199	0.034	0.995	5.011	0.040	0.993
17.5	1.5	7.085	0.007	0.999	6.204	0.015	0.998

[1] $\bar{z}$ and $\bar{z_m}$ are the average values of the price-consumption and price-dividend ratios for the aggregate consumption and market portfolios, respectively.

[2] k_0, k_1, k_{0m}, and k_{1m} are the constants appearing in the approximation equations for the gross returns to the consumption and market portfolios in Equations (6-7).

[3] NA's mean that values are not available for the corresponding combination of γ and ψ.

Table 6: Asset Pricing Implications - Consumption Filtering

γ	ψ	$E(r_m - r_f)$	$E(r_f)$	$\sigma(r_m)$	$\sigma(r_f)$	$\sigma(p-d)$
			SV Gaussian			
7.5	0.5	-0.29	4.45	3.92	2.05	0.01
7.5	1.5	0.00	1.93	2.52	0.68	0.00
10	0.5	-0.42	4.49	3.97	2.05	0.02
10	1.5	0.00	1.91	2.52	0.68	0.00
15	0.5	-0.81	4.59	4.22	2.06	0.02
15	1.5	-0.03	1.84	2.55	0.69	0.00
17.5	0.5	NA	NA	NA	NA	NA
17.5	1.5	NA	NA	NA	NA	NA
			SV DPL			
7.5	0.5	0.06	4.54	4.43	2.18	0.01
7.5	1.5	0.35	1.94	5.61	0.73	0.03
10	0.5	0.17	4.53	4.57	2.18	0.02
10	1.5	0.61	1.89	5.87	0.73	0.03
15	0.5	0.62	4.53	5.47	2.19	0.03
15	1.5	1.48	1.77	7.42	0.73	0.05
17.5	0.5	1.49	4.60	7.89	2.21	0.06
17.5	1.5	3.54	1.67	12.78	0.75	0.10
			IID Gaussian			
7.5	0.5	-0.17	4.42	3.12	1.56	0.01
7.5	1.5	0.13	1.91	2.75	0.52	0.01
10	0.5	-0.23	4.44	3.12	1.56	0.01
10	1.5	0.19	1.88	2.75	0.52	0.01
15	0.5	-0.35	4.47	3.12	1.56	0.01
15	1.5	0.32	1.82	2.75	0.52	0.01
17.5	0.5	-0.42	4.49	3.12	1.56	0.01
17.5	1.5	0.38	1.78	2.75	0.52	0.01
			IID DPL			
7.5	0.5	0.01	4.25	4.22	1.39	0.01
7.5	1.5	0.19	1.82	5.25	0.46	0.03
10	0.5	0.07	4.14	4.22	1.39	0.01
10	1.5	0.34	1.73	5.25	0.46	0.03
15	0.5	0.21	3.93	4.22	1.39	0.01
15	1.5	0.62	1.57	5.25	0.46	0.03
17.5	0.5	0.27	3.82	4.23	1.39	0.01
17.5	1.5	0.77	1.48	5.25	0.46	0.03

[1] The table reports implied expected market risk premium and the risk free rate along with their volatilities, and the volatility of the implied price-dividend ratio for the Gaussian and DPL models for various values of the risk aversion coefficient γ and the intertemporal elasticity of substitution ψ.

[2] NA's mean that values are not available for the corresponding combination of γ and ψ.

Table 7: Maximum Likelihood Parameter Estimates of Dividend Growth Process with Dividend Filtering

SV Model	μ_d	ϕ	σ_e	ρ	σ_d							LogL
Gaussian	0.0087	2.6490	1.5671	0.9507	0.0058							523.3018
	0.0061	1.4416	0.7608	0.0265	0.0005							
DPL	μ_d	ϕ	σ_e	ρ	γ^d_+	γ^d_-	$\gamma^d_+ = \gamma^d_-$	β^d_+	β^d_-	$\beta^d_+ = \beta^d_-$	α^d	LogL
Unrestricted	0.0056	2.9996	1.3489	0.9634	0.0011	0.0018		67.9835	67.9893		1.2443	524.7203
DPL	0.0000	0.0002	0.0000	0.0007	0.0002	0.0003		0.0000	0.0002		0.0000	
Sym. Damp.	0.0056	3.0000	1.3767	0.9619			0.0012			67.9868	1.2794	524.5370
and Scale	0.0000	0.0000	0.0000	0.0269			0.0002			0.0000	0.0000	
Sym. Damp.	0.0056	2.9994	1.4102	0.9610	0.0010	0.0016				67.9700	1.2665	524.6311
	0.0000	0.0001	0.0000	0.0050	0.0004	0.0000				0.0000	0.0000	
Sym. Scale	0.0056	3.0000	1.3767	0.9619			0.0012	67.9868	67.9868		1.2794	524.5379
	0.0000	0.0000	0.0000	0.0000			0.0000	0.0000	0.0002		0.0000	
IID Model	μ_d	ϕ	σ_e	ρ	σ_d							LogL
Gaussian	0.0036	0.6608	0.0091	0.8616	0.0045							528.8095
	0.0036	0.1110	0.0008	0.0513	0.0009							
DPL	μ_d	ϕ	σ_e	ρ	γ^d_+	γ^d_-	$\gamma^d_+ = \gamma^d_-$	β^d_+	β^d_-	$\beta^d_+ = \beta^d_-$	α^d	LogL
Unrestricted	0.0047	2.9262	0.0018	0.8916	0.0001	0.0000		55.9008	55.9530		1.6482	529.9017
DPL	0.0000	0.0000	0.0000	0.0021	0.0001	0.0001		0.0001	0.0001		0.0000	
Sym. Damp.	0.0038	2.9197	0.0017	0.8952			0.0002			55.2986	1.5656	529.5804
and Scale	0.0000	0.0000	0.0000	0.0000			0.0000			0.0000	0.0000	
Sym. Damp.	0.0037	2.8998	0.0019	0.8800	0.0002	0.0001				55.9140	1.5924	530.1220
	0.0000	0.0000	0.0000	0.0023	0.0002	0.0002				0.0000	0.0000	
Sym. Scale	0.0046	2.9197	0.0016	0.9066			0.0003	55.2572	164.7531		1.5223	529.6409
	0.0000	0.7640	0.0000	0.0000			0.0000	0.0000	0.0000		0.0000	

1 Dividend Growth Process: $g_{d,t+1} = \mu_d + \phi x_t + \eta_{d,t+1}$. DPL model: $\eta_{d,t+1} \sim iidDPL(\gamma^d_+, \gamma^d_-, \beta^d_+, \beta^d_-, \alpha^d)$; Gaussian model: $\eta_{d,t+1} \sim iidN(0, \sigma^2_d)$ with state: $x_{t+1} = \rho x_t + e_{t+1}$. $e_{t+1} \sim iidN(0, \sigma^2_e)$ for "iid Model".

2 "Sym. Damp." and "Sym. Scale" refer to the DPL models with "Symmetric Dampening" and "Symmetric Scale". Parameter estimates are reported in the first row, and standard errors in the second.

Table 8: Maximum Likelihood Parameter Estimates of Consumption Growth Process with Dividend Filtering

SV Model	μ_c	σ_c							LogL
Gaussian	0.0067	0.0063							573.5230
	0.0005	0.0004							
DPL	μ_c	γ_+^c	γ_-^c	$\gamma_+^c = \gamma_-^c$	β_+^c	β_-^c	$\beta_+^c = \beta_-^c$	α^c	LogL
Unrestricted	0.0048	0.0032	0.0018		144.0433	53.0835		1.1998	587.4130
DPL	0.0000	0.0000	0.0001		0.0000	0.0285		0.0000	
Sym. Damp.	0.0050			0.0002			20.6503	1.5595	586.6060
and Scale	0.0000			0.0000			0.0032	0.0002	
Sym. Damp.	0.0048	0.0041	0.0052				112.8061	1.1091	587.2645
	0.0000	0.0000	0.0000				0.0000	0.0000	
Sym. Scale	0.0054			0.0001	20.2043	20.9346		1.6056	586.8430
	0.0000			0.0001	0.0998	2.0018		0.1398	
IID Model	μ_c	σ_c							LogL
Gaussian	0.0054	0.0119							473.5237
	0.0009	0.0007							
DPL	μ_c	γ_+^c	γ_-^c	$\gamma_+^c = \gamma_-^c$	β_+^c	β_-^c	$\beta_+^c = \beta_-^c$	α^c	LogL
Unrestricted	0.0052	0.0078	0.0024		211.5519	90.5366		1.1514	581.2584
DPL	0.0000	0.0000	0.0012		0.0277	0.0035		0.0036	
Sym. Damp.	0.0050			0.0010			100.0069	1.3710	580.9752
and Scale	0.0000			0.0000			0.0000	0.0783	
Sym. Damp.	0.0046	0.0009	0.0008				102.5859	1.3970	580.5284
	0.0000	0.0002	0.0000				0.0391	0.0000	
Sym. Scale	0.0052			0.0010	107.4070	106.2566		1.3720	580.4390
	0.0000			0.0001	0.0002	0.0004		0.0000	

1 Consumption Growth Process: $g_{c,t+1} = \mu_c + x_t + \eta_{c,t+1}$; DPL model: $\eta_{c,t+1} \sim iidDPL(\gamma_+^c, \gamma_-^c, \beta_+^c, \beta_-^c, \alpha^c)$. Gaussian model: $\eta_{c,t+1} \sim iidN(0, \sigma_c^2)$. Assuming consumption growth process shares the same DPL structure with dividends growth process.

2 "Sym. Damp." and "Sym. Scale" refer to the DPL models with "Symmetric Dampening" and "Symmetric Scale". Parameter estimates are reported in the first row, and standard errors in the second.

Table 9: Parameters Determining Price-Consumption and Price-Dividend Ratios with Dividend Filtering

γ	ψ	$\bar{z}$	k_0	k_1	$\bar{z_m}$	k_{0m}	k_{1m}
SV Gaussian							
2.5	0.5	4.871	0.045	0.992	4.948	0.042	0.993
2.5	1.5	8.332	0.002	1.000	8.500	0.002	1.000
3	0.5	4.927	0.043	0.993	4.837	0.046	0.992
3	1.5	7.777	0.004	1.000	6.225	0.014	0.998
3.5	0.5	4.995	0.040	0.993	4.724	0.050	0.991
3.5	1.5	7.378	0.005	0.999	5.535	0.026	0.996
4	0.5	5.088	0.037	0.994	4.594	0.056	0.990
4	1.5	7.031	0.007	0.999	5.050	0.039	0.994
SV DPL							
2	0.5	5.211	0.034	0.995	5.077	0.038	0.994
2	1.5	6.932	0.008	0.999	6.071	0.016	0.998
3	0.5	5.338	0.030	0.995	4.880	0.044	0.992
3	1.5	6.711	0.009	0.999	5.331	0.031	0.995
4	0.5	5.529	0.026	0.996	4.680	0.052	0.991
4	1.5	6.499	0.011	0.998	4.878	0.044	0.992
4	0.5	5.972	0.018	0.997	4.411	0.065	0.988
4	1.5	6.265	0.014	0.998	4.480	0.061	0.989
IID Gaussian							
2.5	0.5	5.460	0.027	0.996	5.686	0.023	0.997
2.5	1.5	6.646	0.010	0.999	7.816	0.004	1.000
3	0.5	5.766	0.021	0.997	6.602	0.010	0.999
3	1.5	6.402	0.012	0.998	5.280	0.032	0.995
3.5	0.5	6.228	0.014	0.998	3.696	0.114	0.976
3.5	1.5	6.208	0.014	0.998	3.527	0.130	0.971
4	0.5	7.160	0.006	0.999	2.972	0.195	0.951
4	1.5	6.047	0.017	0.998	3.186	0.167	0.960
IID DPL							
2.5	0.5	5.007	0.040	0.993	4.862	0.045	0.992
2.5	1.5	7.698	0.004	1.000	6.487	0.011	0.998
3	0.5	5.022	0.039	0.993	4.854	0.045	0.992
3	1.5	7.623	0.004	1.000	6.353	0.013	0.998
3.5	0.5	5.037	0.039	0.994	4.847	0.046	0.992
3.5	1.5	7.552	0.004	0.999	6.235	0.014	0.998
4	0.5	5.053	0.038	0.994	4.839	0.046	0.992
4	1.5	7.487	0.005	0.999	6.130	0.015	0.998

[1] $\bar{z}$ and $\bar{z_m}$ are the average values of the price-consumption and price-dividend ratios for the aggregate consumption and market portfolios, respectively.

[2] k_0, k_1, k_{0m}, and k_{1m} are the constants appearing in the approximate equations for the gross returns to the consumption and market portfolios in Equations (6-7).

Table 10: Asset Pricing Implications - Dividend Filtering

γ	ψ	$E(r_m - r_f)$	$E(r_f)$	$\sigma(r_m)$	$\sigma(r_f)$	$\sigma(p-d)$
SV Gaussian						
2.5	0.5	0.06	6.25	5.66	2.69	0.08
2.5	1.5	1.23	2.33	17.10	0.92	0.27
3	0.5	0.25	6.39	5.84	2.70	0.08
3	1.5	2.02	2.26	17.21	0.94	0.27
3.5	0.5	0.47	6.55	6.30	2.72	0.08
3.5	1.5	2.89	2.17	17.76	0.97	0.26
4	0.5	0.76	6.75	7.17	2.76	0.08
4	1.5	3.98	2.05	18.96	1.02	0.26
SV DPL						
2.5	0.5	-0.02	4.75	8.87	2.68	0.15
2.5	1.5	1.45	1.71	21.61	0.94	0.39
3	0.5	0.37	4.90	9.01	2.70	0.15
3	1.5	2.59	1.58	21.36	0.97	0.37
3.5	0.5	0.82	5.11	9.73	2.76	0.15
3.5	1.5	3.83	1.44	21.88	1.02	0.36
4	0.5	1.57	5.49	11.85	2.89	0.16
4	1.5	5.50	1.25	23.58	1.10	0.35
IID Gaussian						
2.5	0.5	-2.58	5.37	18.64	7.17	0.17
2.5	1.5	-0.02	1.62	2.55	2.39	0.00
3	0.5	-3.77	5.75	18.88	7.17	0.17
3	1.5	-0.03	1.46	2.55	2.39	0.00
3.5	0.5	-4.02	6.14	16.59	7.17	0.15
3.5	1.5	-0.03	1.30	2.55	2.39	0.00
4	0.5	-4.19	6.55	14.75	7.17	0.13
4	1.5	-0.03	1.14	2.55	2.39	0.00
IID DPL						
2.5	0.5	-0.02	4.63	3.57	1.56	0.03
2.5	1.5	0.15	1.98	7.56	0.52	0.08
3	0.5	0.03	4.61	3.57	1.56	0.03
3	1.5	0.28	1.95	7.54	0.52	0.08
3.5	0.5	0.07	4.59	3.57	1.56	0.03
3.5	1.5	0.40	1.91	7.53	0.52	0.08
4	0.5	0.12	4.57	3.57	1.56	0.03
4	1.5	0.52	1.88	7.51	0.52	0.08

The table reports implied expected market risk premium and the risk free rate along with their volatilities, and the volatility of the implied price-dividend ratio for various values of the risk aversion coefficient γ and the intertemporal elasticity of substitution ψ.