Second Mid-Term Exam

Intermediate Macroeconomics

Spring 2006

Name: Answer key

Please answer all questions. Be sure to explain your math with a few words. That way algebraic errors will be less costly.

1. Please provide one-sentence explanations of the following terms:

a) Shadow price: The increase in profits caused by having one extra unit of a constrained resource.

b) increasing returns to scale: When a doubling of ~~use~~ inputs leads to more than a doubling of output.

c) isoquant: All combinations of inputs that produce the same fixed level of output.

d) comparative statics: The analysis of how changes in parameter values alter equilibrium choices, say of input use or output.

e) $F_{LK}>0$: The ~~co~~ derivative indicates L and K are complements in production.

2. Prove algebraically that the short-run marginal cost curve passes through the short-run average variable cost curve at its minimum point.

Define: $AVC = \frac{c(y)}{y}$.

Take derivative and set to zero to find minimum:

$$\frac{c'(y)}{y} - \frac{c(y)}{y^2} = 0$$

$$\Rightarrow c'(y) = \frac{c(y)}{y} \Rightarrow MC = AVC.$$

3. Suppose that in the short-run a perfectly competitive firm faces $p=64$ and has a total cost function $c(q) = 2q^3 + 10q + 28$. At what level of output will the firm produce? What is the firm's profits at this level of production?

$MC = 6q^2 + 10$.

Set $MC = P$. $\Rightarrow 6q^2 + 10 = 64$

$$\Rightarrow q^* = \sqrt{\frac{54}{6}} = 3$$

Profits are $64(3) - (2(3)^3 + 10(3) + 28)$

$= 192 - 56$

$= 136$

4. Suppose short-run output is given by $y = L^{\alpha} + \bar{K}$.

a) What is the equation of the supply curve?

$$\pi = p(L^{\alpha} + \bar{K}) - wL - r\bar{K}$$

$$\frac{d\pi}{dL} = \alpha p L^{\alpha-1} - w = 0 \Rightarrow L = \left(\frac{\alpha p}{w}\right)^{1/1-\alpha}$$

$$\Rightarrow y = \left(\frac{\alpha p}{w}\right)^{\alpha/1-\alpha} + \bar{K}$$

b) How does the supply curve shift if the quantity of capital changes?

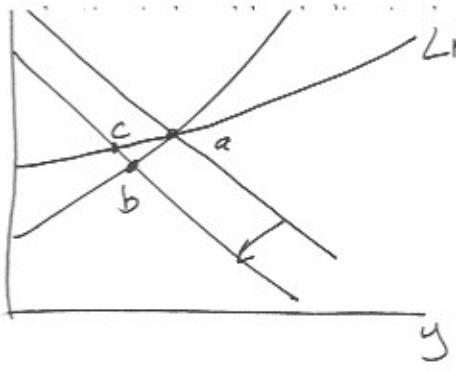
It shifts right by one unit for each one unit increase in $\bar{K}$.

c) What is the shadow price of capital?

$$\pi = p\left[\left(\frac{\alpha p}{w}\right)^{\alpha/1-\alpha} + \bar{K}\right] - wL - r\bar{K}$$

$$\frac{d\pi}{d\bar{K}} = p - r$$ ~ This is the shadow price.

5. Explain in words, perhaps with the aid of a diagram, why the short-run price



... demand causes an output ... prone to b. This induces firms to exit, or cut back on K, causing the long-run response to be at c.

6. Prove algebraically that in the long-run cost-minimization problem the slope of the isocost line must equal the slope of the isoquant.

$$\pi = pF(K,L) - wK - rL$$

Two foc's are

$$pf_L = w$$

$$pf_K = r.$$

Take ratios

$$\frac{f_L}{F_K} = \frac{w}{r}$$

Left hand side is the negative of the slope of the isoquant. Right hand side is the negative of the slope of the isocost line.

7. Suppose long-run output is given by $y = \sqrt{L + K}$, that w=3, r=4, and p=12.

a) What are the profit-maximizing quantities of inputs?

L and K are perfect substitutes and L is cheaper, so only L is used.

Hence, $\pi = p\sqrt{L} - wL$

$\frac{d\pi}{dL} = \frac{p}{2\sqrt{L}} - w = 0 \Rightarrow L^* = \left(\frac{p}{2w}\right)^2 = 4$; $K^* = 0$

b) What is the profit-maximizing level of output?

$y^* = \sqrt{L^*}$

$= \sqrt{4}$

$= 2$

c) What are profits?

$\pi^* = p\sqrt{L^*} - wL^*$

$= 24 - 12$

$= 12$

d) Imagine that the firm is required to pay a license of \$15 per period to operate. How does this affect the optimal quantity of K and L?

This would make $\pi^* = -3$, so it is better not to produce. Hence $L^* = K^* = 0$