MAC 2313 (Calculus III)
Test 1, Thursday September 28, 2006

Name: **PID:**

Remember that no documents or graphing calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve any of the points assigned to any question. *You will not get any credit by just writing down the answer to any of the problems.* **Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.**

1. [10] Let m be a real number. Consider the surface given by the equation: $x^2 + y^2 + z^2 - 2x + 2my + 4z + 9 = 0$. For which values of m is that surface a sphere? a point?

2. [10] Plot the two points $A(1, 2, -2)$, and $B(-1, 2, -3)$, and sketch the vector $\mathbf{v} = \overrightarrow{AB}$. What is $||\mathbf{v}||$? Find to the nearest degree the angle that $\mathbf{v}$ makes with the positive $x-$axis.

3. [10] If $\mathbf{u}= \vec{i} + 2\vec{j} + \vec{k}$, and $\mathbf{b}=- \vec{i} + \vec{j} + 3\vec{k}$, find the component of $\mathbf{u}$ that is parallel to $\mathbf{b}$, and the component of $\mathbf{b}$ that is parallel to $\mathbf{u}$.

4. [10] Show that in 3-space the distance d from a point P to the line L through points A and B can be expressed as $d = \frac{||\overrightarrow{AP} \times \overrightarrow{AB}||}{||\overrightarrow{AB}||}$.

5. [10] If θ is the angle between $\mathbf{u}$=$-2\vec{i} + \vec{j} + \vec{k}$, and $\mathbf{v}$=$\vec{i} - 2\vec{j} + \vec{k}$, find $\sin\theta$. Is θ acute or obtuse?

6. [10] Show that the two lines $L_1 : x = 1 - t, \quad y = -1, \quad z = t$, and $L_2 : x = 1, \quad y = t, \quad z = 2 + 2t$ intersect, and find their point of intersection.

7. [12] a) Convert $(3/2, \sqrt{3}/2, -1)$ from rectangular coordinates to spherical coordinates. b) Convert the equation $x^2 - y^2 - z^2 = 1$ from rectangular coordinates to cylindrical coordinates.

8. [20] If $r(t) = e^{-t}\overrightarrow{i} + \sin(1 - e^{-t})\overrightarrow{j} + \cos(1 - e^{-t})\overrightarrow{k}$, find **T**, **N**, **B**, as well as the equations of the osculating, and the rectifying planes at time $t = 0$.

9. [10] If $r(t) = \sqrt{3}e^{t}\overrightarrow{i} + \sin(e^{t})\overrightarrow{j} + \cos(e^{t})\overrightarrow{k}$, find an arc length parametrization of the curve that has the same orientation as the given curve, and has $t = 0$ as the reference point.