

Theory of Algorithms. Spring 2000. Homework Assignment 8.

Section 7.2 The Equivalence of NPDAs and Context-Free Grammars.

(1) Let G be the following context-free grammar, with $V = \{S, B, C\}$, $T = \{a, b, c\}$.

$$S \rightarrow aSbbB \mid C \mid \lambda$$

$$B \rightarrow b \mid \lambda$$

$$C \rightarrow Cc \mid c$$

(a) Find an npda M such that $L(M) = L(G)$.

(b) Give a left-most derivation from G of the string: $aaccbbbbb$.

(c) Give the corresponding sequence of instantaneous descriptions for M .

(2) Let M be the following npda, with $\Gamma = \{S, 0, 1, z\}$, $F = \{q_f\}$.

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$

$$\delta(q_1, a, S) = \{(q_1, 11s)\}$$

$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$

$$\delta(q_1, a, 0) = \{(q_1, \lambda), (q_1, 010)\}$$

$$\delta(q_1, b, 1) = \{(q_1, \lambda)\}$$

$$\delta(q_1, b, S) = \{(q_1, \lambda), (q_1, 0S)\}$$

$$\delta(q_1, \lambda, 0) = \{(q_1, 000), (q_1, \lambda)\}$$

$$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$$

Find a context-free grammar G such that $L(G) = L(M)$.

(3) (OPTIONAL) Let M' be the following npda, with $Q' = \{q'_0, q'_1, q'_2\}$, $F' = \{q'_1, q'_2\}$, $\Sigma' = \{a, b\}$, and $\Gamma' = \{z', a, b\}$.

$$\delta'(q'_0, a, z') = \{(q'_0, az'), (q'_2, \lambda)\}$$

$$\delta'(q'_0, a, a) = \{(q'_0, aa), (q'_1, \lambda)\}$$

$$\delta'(q'_0, b, a) = \{(q'_0, ba)\}$$

$$\delta'(q'_1, b, b) = \{(q'_1, \lambda)\}$$

$$\delta'(q'_1, a, a) = \{(q'_1, \lambda)\}$$

$$\delta'(q'_2, a, a) = \{(q'_2, a)\}$$

Find an npda M such that $L(M) = L(M')$ and M is in pre-standard form.

(4) Let M be the following npda with $F = \{q_f\}$.

$$\delta(q_0, a, z) = \{(q_0, az)\}$$

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, a, b) = \{(q_0, ab)\}$$

$$\delta(q_0, b, z) = \{(q_0, bz)\}$$

$$\delta(q_0, b, a) = \{(q_0, ba)\}$$

$$\delta(q_0, b, b) = \{(q_0, bb)\}$$

$$\delta(q_1, a, a) = \{(q_1, \lambda)\}$$

$$\delta(q_1, b, b) = \{(q_1, \lambda)\}$$

$$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$$

$$\delta(q_0, \lambda, a) = \{(q_1, a)\}$$

$$\delta(q_0, \lambda, b) = \{(q_1, b)\}$$

(a) Find an npda \bar{M} such that $L(\bar{M}) = L(M)$, and \bar{M} is in standard reduced form. Use the clause-template notation that I used in the example in the notes.

(b) Give the sequences of instantaneous descriptions for M and \bar{M} that show that M and \bar{M} accept the string: $abaaba$.