Theory of Algorithms. Spring 2000. Section 2. Quiz 8

SOLUTIONS.

(1) Let G be the following context-free grammar, with $V = \{S, A\}$, $T = \{a, b\}$.

$$S \to ASb \mid Ab$$
$$A \to a \mid \lambda$$

(a) Find an npda M such that $L(M) = L(G)$.

SOLUTION: $F = \{q_f\}$.

$$\begin{aligned}
\delta(q_0, \lambda, z) &= \{(q_1, Sz)\} \\
\delta(q_1, \lambda, S) &= \{(q_1, ASb), (q_1, Ab)\} \\
\delta(q_1, \lambda, A) &= \{(q_1, a), (q_1, \lambda)\} \\
\delta(q_1, a, a) &= \{(q_1, \lambda)\} \\
\delta(q_1, b, b) &= \{(q_1, \lambda)\} \\
\delta(q_1, \lambda, z) &= \{(q_f, \lambda)\}
\end{aligned}$$

(b) Give a **left-most** derivation from G of the string: $abbb$.
(WARNING: Make sure your derivation is left-most.)

SOLUTION:

$S \Rightarrow ASb \Rightarrow aSb \Rightarrow aASbb \Rightarrow aSbb \Rightarrow aAbbb \Rightarrow abbb.$

(c) Give the corresponding sequence of instantaneous descriptions for M.

SOLUTION:

$(q_0, abbb, z) \vdash (q_1, abbb, Sz) \vdash (q_1, abbb, ASbz) \vdash (q_1, abbb, aSbz) \vdash (q_1, bbb, Sbz) \vdash (q_1, bbb, ASbbz) \vdash (q_1, bbb, Sbbz) \vdash (q_1, bbb, Abbbz) \vdash (q_1, bbb, bbbz) \vdash (q_1, bb, bbz) \vdash (q_1, b, bz) \vdash (q_1, \lambda, z) \vdash (q_f, \lambda, \lambda)$

THERE IS ANOTHER PROBLEM ON THE OTHER SIDE OF THE PAPER!

(2) Let M be the npda with $Q = \{q_0, q_1, q_2, q_f\}$, $F = \{q_f\}$, $\Gamma = \{1, z\}$, $\Sigma = \{a, b, c\}$, and:

$$\delta(q_0, a, z) = \{(q_0, 1z)\} \qquad \delta(q_1, b, 1) = \{(q_1, \lambda)\} \qquad \delta(q_1, c, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_0, a, 1) = \{(q_0, 11)\} \qquad \delta(q_2, \lambda, 1) = \{(q_1, \lambda)\}$$
$$\delta(q_0, b, 1) = \{(q_1, \lambda)\} \qquad \delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$$

(a) Find an npda $\bar{M}$ such that $L(\bar{M}) = L(M)$, and $\bar{M}$ is in standard reduced form. Use the clause-template notation. You only have to write down the $\bar{\delta}$-clauses. Don't bother with $\bar{Q}$ etc.

SOLUTION:

$$\bar{\delta}(\bar{q}_0, \lambda, \bar{z}) = \{(\bar{q}_1, (q_0, z, q_f)\bar{z})\}$$
$$\bar{\delta}(\bar{q}_1, \lambda, \bar{z}) = \{(\bar{q}_f, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, b, (q_0, 1, q_1)\big) = \{(\bar{q}_1, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, b, (q_1, 1, q_1)\big) = \{(\bar{q}_1, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, \lambda, (q_1, z, q_f)\big) = \{(\bar{q}_1, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, c, (q_1, 1, q_2)\big) = \{(\bar{q}_1, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, \lambda, (q_2, 1, q_1)\big) = \{(\bar{q}_1, \lambda)\}$$
$$\bar{\delta}\big(\bar{q}_1, a, (q_0, z, q_x)\big) = \Big\{ \Big(\bar{q}_1, (q_0, 1, q_y)(q_y, z, q_x)\Big)\Big\}.$$
$$\bar{\delta}\big(\bar{q}_1, a, (q_0, 1, q_x)\big) = \Big\{ \Big(\bar{q}_1, (q_0, 1, q_y)(q_y, z, q_x)\Big)\Big\}.$$

(b) Give a sequences of instantaneous descriptions for M that shows that M accepts the string: $aaabc$.

SOLUTION:

$(q_0, aaabc, z) \vdash (q_0, aabc, 1z) \vdash (q_0, abc, 11z) \vdash (q_0, bc, 111z) \vdash (q_1, c, 11z) \vdash (q_2, \lambda, 1z) \vdash (q_1, \lambda, z) \vdash (q_f, \lambda, \lambda)$

(c) Give the corresponding sequences of instantaneous descriptions for $\bar{M}$.

SOLUTION:

$(\bar{q}_0, aaabc, \bar{z}) \vdash (\bar{q}_1, aaabc, (q_0, z, q_f)\bar{z}) \vdash (\bar{q}_1, aabc, (q_0, 1, q_1)(q_1, z, q_f)\bar{z}) \vdash$
$(\bar{q}_1, abc, (q_0, 1, q_2)(q_2, 1, q_1)(q_1, z, q_f)\bar{z}) \vdash (\bar{q}_1, bc, (q_0, 1, q_1)(q_1, 1, q_2)(q_2, 1, q_1)(q_1, z, q_f)\bar{z}) \vdash$
$(\bar{q}_1, c, (q_1, 1, q_2)(q_2, 1, q_1)(q_1, z, q_f)\bar{z}) \vdash (\bar{q}_1, \lambda, (q_2, 1, q_1)(q_1, z, q_f)\bar{z}) \vdash (\bar{q}_1, \lambda, (q_1, z, q_f)\bar{z}) \vdash (\bar{q}_1, \lambda, \bar{z}) \vdash$
$(\bar{q}_f, \lambda, \lambda)$