

1) Use the Wronskian to decide whether the functions $\{x + 2, x^2 - 1\}$ are LI in P_3 . Then comment briefly on your reasoning.

2) Let $S \subseteq R^3$ be the set of vectors of the form $[a + 2b + c, 2a + 4b, a + 2b]^T$. Find a basis for S , check whether it really is a basis, and comment.

3) Answer True or False:

The columns of a nonsingular matrix are always linearly independent.

If L is a list of n vectors, then $\dim(\text{span } L) = n$.

If A is a singular 4×4 matrix, then its nullity is at least 1.

R^2 is a subspace of R^3 .

If L is a list of 3 L.I. functions in P_3 , then $\text{span}(L) = P_3$.

4) Choose ONE of these to prove (on the back is OK).

a) If U and V are subspaces of a vector space W , then $U \cap V$ is too.

b) [This is part of Thm.3.3.2] Suppose L is a linearly dependent list of vectors, and $\mathbf{v} \in \text{span}(L)$. Then \mathbf{v} is a linear combination of the vectors in L in at least *two* different ways.

Remarks and Answers: The problems were worth 15 points each. The average was about 40/60 (similar to Quiz 2). I wrote your Quiz 1-4 total “ Σ ” in the upper right corner, with your approximate semester grade so far. The A-’s started at about 200, and each letter is 25 points. Later I will include your HW, and will replace your lowest quiz score with your average MHW grade.

1) $W = \det \begin{pmatrix} x+2 & x^2-1 \\ 1 & 2x \end{pmatrix} = x^2 + 4x + 1 \neq 0$. So, they are LI.

2) Your final answer should be a basis of S ; a list of LI vectors in R^3 . Not a matrix, not a scalar, etc. This problem is very similar to HW 3.4.7 (factor out the a, b, c), but since the 3 vectors you get that way are LD, you must remove one (-3 points if you didn’t).

$$\begin{pmatrix} a+2b+c \\ 2a+4b \\ a+2b \end{pmatrix} = a \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + b \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

So, one basis for S is $\{[1 \ 2 \ 1]^T, [1 \ 0 \ 0]^T\}$.

3) TFFT

4) See lectures and the text.