

Problems for the week Sept. 13 - Sept. 20

1. Find the solutions in $(0, \infty)$ of the system:

$$\begin{cases} x + y + z = 9 \\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 \end{cases}$$

2. Let \mathbf{C}_1 and \mathbf{C}_2 be concentric circles, with \mathbf{C}_2 in the interior of \mathbf{C}_1 . From a point A on \mathbf{C}_1 one draws the tangent AB to \mathbf{C}_2 ($B \in \mathbf{C}_2$). Let C be the second point of intersection of AB and \mathbf{C}_1 , and let D be the midpoint of AB . A line passing through A intersects \mathbf{C}_2 at E and F in such a way that the perpendicular bisectors of DE and CF intersect at a point M on AB . Find, with proof, the ratio AM/MC .

3. A computer screen shows a 98×98 chessboard, colored in the usual way. One can select with a mouse any rectangle with sides on the lines of the chessboard and click the mouse button: as a result, the colors in the selected rectangle switch (black becomes white, white becomes black). Find, with proof, the minimum number of mouse clicks needed to make the chessboard all one color.