

## Math Circle Problems for week 5 - More Mathematical Induction

1. Prove by induction the arithmetic–geometric mean inequality:  
for any positive integer  $n$  and for any positive (real) numbers  $a_1, a_2, \dots, a_n$ ,

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}.$$

2. Apply the arithmetic–geometric mean inequality to also deduce the  
geometric–harmonic mean inequality

$$\sqrt[n]{a_1 a_2 \dots a_n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}.$$