Math Circle Problems for week 4 - Mathematical Induction

1. Show that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$

2. Show that $1^3 + 2^3 + ... + n^3 = n^2(n+1)^2/4$

3. Show that for any fixed integer $m > 0$,

$$\frac{m!}{0!} + \frac{(m+1)!}{1!} + ... + \frac{(m+n)!}{n!} = \frac{(m+n+1)!}{n!(m+1)!}$$

4.Show that $2^{3^n} + 1$ is divisible by 3^{n+1}.

5. Show that $3^{2n+2} + 8n - 9$ is divisible by 16.

6. Prove that among any 2^{n+1} natural numbers there are 2^n numbers whose sum is divisible by 2^n.