

Math Circle Problems for week 1 - Number Theory

1. Find all positive integer solutions x, y of the equation $x^2 - y^2 = 303$.
2. Is it true that for any even number n , the number $(n + 1)(n + 3)(n + 5)(n + 7)(n + 9)$ is divisible by 165? Justify your answer.
3. Ten thousand sailors are arranged around the edge of their ship. They hold, in order, the numbers 1, 2, 3, ..., 10000. Starting the count with number 1, every other sailor is pushed overboard (by the crazy captain of the ship) until they are all gone. Which is the number of the last survivor?
(*Hint:* Do the problem first for small numbers of sailors and try to see patterns.)