

CONTEST PROBLEMS

1a) Let the altitudes from the vertices A , B and C of an acute triangle ABC intersect the sides BC , AC and AB respectively in P, Q and R (these points are called feet of the altitudes). Prove that AP , BQ and CR are interior angle bisectors of the triangle PQR

b) Construct geometrically (by straight edge and compass) or analytically (in coordinates) the vertices A , B , and C of an (acute) triangle ABC if you are given the feet P, Q , and R of the altitudes of that triangle.

2a) Construct geometrically or analytically the vertices A , B , and C of a triangle ABC if you are given the images O_1, O_2 , and O_3 of the circum-center O of that triangle under reflections across the sides BC , CA , and AB respectively.

b) Show that the triangle O_1, O_2, O_3 is congruent to ABC and that O is its orthocenter. This could help also for part a .

3) The measure of the angle at the vertex B of the triangle ABC is 60° . Assume the interior angle bisectors at A, C respectively intersect the opposite sides at D, E respectively. Suppose AD and CE intersect at O . Prove that $|OD| = |OE|$.

4) Let $ABCD$ be a convex quadrilateral without parallel sides and E and F be the points of intersections of the lines AB, CD and AD, BC respectively. Suppose that the diagonals AC and BD intersect the line EF in P and Q respectively. Prove that

$$\frac{PF}{PE} = \frac{QF}{QE}$$

5) On the sides of an arbitrary triangle ABC , exterior to it, construct isosceles triangles BCA_1 , ACB_1 and ABC_1 with $|A_1B| = |A_1C|$, $|B_1A| = |B_1C|$, $|C_1A| = |C_1B|$. Let angles at the vertices A_1, B_1, C_1 be equal to α, β, γ respectively. Prove that if $\alpha + \beta + \gamma = 360^\circ$,

then the angles of the triangle $A_1B_1C_1$ are equal to $\frac{1}{2}\alpha$, $\frac{1}{2}\beta$ and $\frac{1}{2}\gamma$. (In particular they do not depend on the shape of ABC .)

6) Let M and N be points on the parallel sides AB and CD respectively of the parallelogram $ABCD$. Suppose that AN intersects DM in a point P , BN intersects CM in a point Q , and the diagonal AC intersects the diagonal BD in R . Prove that P, Q and R are collinear.

7) Let $ABCD$ be a parallelogram and M, N, K and L are on the sides AD, BC, AB and CD respectively, such that MN is parallel to AB and KL is parallel to AC . Let O is the intersection point of MN and LK . Prove that AN, CK and DO meet at one point.

8) A line passing through the vertex C of an equilateral triangle ABC is given. Denote by K and M the orthogonal projections of the vertices A and B on that line, and by P - the midpoint of the segment $[AB]$. Prove that the triangle KMP is also equilateral.

*Deadline for solutions is **June 15, 2005**. Please, submit the solutions with address for correspondence and e-mail to:*

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