Homework Assignment #3

9.8: Use the observation following Theorem 9.2 to carry out a quick calculation of the determinant of each of the following matrices:

$$a)\begin{pmatrix}1&1&1\\1&4&2\\1&4&3\end{pmatrix}\qquad b)\begin{pmatrix}1&1&1\\0&4&5\\1&9&6\end{pmatrix}.$$

We row reduce the first matrix by first subtracting the first row from the second and third rows, obtaining $\begin{pmatrix}1&1&1\\0&3&1\\0&3&2\end{pmatrix}$, and then subtact the second row from the first, obtaining $\begin{pmatrix}1&1&1\\0&3&1\\0&0&1\end{pmatrix}$. As with all row-echelon matrices, we now find the determinant by multiplying the diagonal terms. Thus the matrix in (a) has determinant 3.

For the second matrix, first subtract row 1 from row 3, and then subtract twice row 2 from row 3, obtaining $\begin{pmatrix}1&1&1\\0&4&5\\0&0&-5\end{pmatrix}$. This has determinant -20.

9.13 Use Cramer's rule to solve the following system of equations:

$$a)\ \begin{aligned}5x_1+x_2&=3\\2x_1-x_2&=4;\end{aligned}\qquad b)\ \begin{aligned}2x_1-3x_2&=2\\4x_1-6x_2+x_3&=7\\x_1+10x_2&=1.\end{aligned}$$

For (a), $\det A=-5-2=-7$, and we find $x_1=(-3-4)/(-7)=1$ and $x_2=(20-6)/(-7)=-2$.

For (b), $\det A=-(20+3)=-23$, and we find $x_1=-23/-23=1$, $x_2=0/-23=0$ and $x_3=-69/-23=3$.

9.17 If we introduce tax rate t and let the consumption function depend on after-tax income, $C=b(Y-tY)$, then system (10) becomes

$$\begin{aligned}(1-t)sY+ar&=I^o+G\\mY-hr&=M_s-M^o\end{aligned}$$

Use Cramer's rule to see how the equilibrium Y and r are affected by the tax rate t.

Here $\Delta=\det A=-[(1-t)sh+am]$, and then Cramer's rule implies $Y=-[h(I^o+G)+a(M_s-M^o)]/\Delta$ and $r=[(1-t)s(M_s-M^o)-m(I^o+G)]/\Delta$. Thus

$$\begin{aligned}Y&=\frac{h(I^o+G)+a(M_s-M^o)}{(1-t)sh+am}\\r&=\frac{m(I^o+G)-(1-t)s(M_s-M^o)}{(1-t)sh+am}\end{aligned}$$

We assume $h(I^o+G)+a(M_s-M^o)\geq 0$ and $m(I^o+G)-(1-t)s(M_s-M^o)\geq 0$, so that our answer is economically sensible. Now compute

$$\frac{\partial Y}{\partial t}=sh\frac{h(I^o+G)+a(M_s-M^o)}{[(1-t)sh+am]^2}>0$$

and

$$\frac{\partial r}{\partial t} = \frac{s(M_s - M^o)}{(1-t)sh + am} + sh\frac{m(I^o + G) - (1-t)s(M_s - M^o)}{[(1-t)sh + am]^2}$$
$$= sm\frac{h(I^o + G) + a(M_s - M^o)}{[(1-t)sh + am]^2} > 0.$$

Both Y and r are increasing in t.

10.12 For each of the following pairs of vectors, first determine whether the angle between them is acute, obtuse, or right and then calculate this angle:

a) $\mathbf{u} = (1,0)$,	$\mathbf{v} = (2,2)$;	*b*) $\mathbf{u} = (4,1)$,	$\mathbf{v} = (2,-8)$;
c) $\mathbf{u} = (1,1,0)$,	$\mathbf{v} = (1,2,1)$;	*d*) $\mathbf{u} = (1,-1,0)$,	$\mathbf{v} = (1,2,1)$;
e) $\mathbf{u} = (1,0,0,0,0)$,	$\mathbf{v} = (1,1,1,1,1)$.		

In each case, $\cos\theta = \mathbf{u}\cdot\mathbf{v}/(\|\mathbf{u}\|\,\|\mathbf{v}\|)$. For (a), $\cos\theta = 2/2\sqrt{2} = 1/\sqrt{2}$, so the angle is $\theta = 45^o$ (acute). For (b) $\cos\theta = 0$, so the angle is $\theta = 90^o$ (right). For (c), $\cos\theta = 3/\sqrt{2}\sqrt{6} = \sqrt{3}/2$, so the angle is $\theta = 30^o$ (acute). For (d), $\cos\theta = -1/\sqrt{2}\sqrt{6} = -1/2\sqrt{3}$, so the angle is $\theta = 106.8^o$ (obtuse). For (e), $\cos\theta = 1/\sqrt{5}$, so the angle is $\theta = 63.4^o$ (acute).

10.15 Prove that $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 - 2\mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2$.

$\|\mathbf{u} - \mathbf{v}\|^2 = (\mathbf{u} - \mathbf{v})\cdot(\mathbf{u} - \mathbf{v}) = \mathbf{u}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{u} - \mathbf{v}\cdot\mathbf{v} = \|vu\|^2 - 2\mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2$, where we use the fact that $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$.