Homework Assignment #1

6.2 In Missouri, federal income taxes are deducted from income before calculating state taxes. Write out and solve the system of equations which describes the state and federal taxes and charitable contributions of the firm in Example 1 if it were based in Missouri.

Note that I've corrected the question. The formulas for after-tax profits and for federal tax are unchanged from Example 1. The difference is in the computation of state tax S. State tax is now calculated based on income of \$100,000, minus charitable contributions C and federal tax F. Since the state income tax rate is 5%, the state tax is $S = .05(100,000 - C - F)$. This yields the following system

$$
\begin{aligned}
C + 0.1S + 0.1F &= 10,000 \\
0.05C + S + 0.05F &= 5,000 \\
0.4C + 0.4S + F &= 40,000.
\end{aligned}
$$

After some calculations, the solution is $C = 6070$, $F = 36,422$, and $S = 2875$.

6.3 The economy on the island of Bacchus produces only grapes and wine. The production of 1 pound of grapes requires 1/2 pound of grapes, 1 laborer, and no wine. The production of 1 liter of wine requires 1/2 pound of grapes, 1 laborer, and 1/4 liter of wine. The island has 10 laborers who all together demand 1 pound of grapes and 3 liters of wine for their own consumption. Write out the input-output system for the economy of this island. Can you solve it?

Let g and w respectively denote the amount of grapes and wine produced. Demand for grapes from wine production is $w/2$, demand from grape production is $g/2$ and demand for consumption is 1. Since demand must equal supply, $g = g/2 + w/2 + 1$. For wine, the corresponding equation is $w = w/4 + 3$. Finally, for labor, $10 = w + g$. This yields the following input-output system:

$$
\begin{aligned}
g/2 - w/2 &= 1 \\
3w/4 &= 3 \\
w + g &= 10
\end{aligned}
$$

This is easily solved since the second equation implies $w = 4$. Then the third equation yields $g = 6$, and we have merely to verify that the first equation is also satisfied (otherwise there would be no solution).

6.7 Consider the IS-LM model of Example 4 with no fiscal policy ($G = 0$). Suppose that $M_s = M^o$; that is, the intercept of the LM curve is 0. Suppose that $I^o = 1000$, $s = 0.2$, $h = 1500$, $a = 2000$, and $m = 0.16$. Write out the explicit IS-LM system of equations. Solve them for the equilibrium GNP Y and the interest rate r.

The IS-LM system of Example 4 is:

$$
\begin{aligned}
sY + ar &= I^o + G \\
mY - hr &= M_s - M^o.
\end{aligned}
$$

Substituting the values above, we obtain:

$$
\begin{aligned}
.2Y + 2000r &= 1000 \\
.16Y - 1500r &= 0.
\end{aligned}
$$

We immediately find that $Y = 9375r$ from the second equation. Substituting in the first equation yields $1875r + 2000r = 1000$. Thus $r = 1000/3875 = .258$ and $Y = 2419$ (approximately).

7.6 Consider the general IS-LM model with no fiscal policy in Chapter 6. Suppose that $M_s = M^o$; that is, the intercept of the LM-curve is 0.

i) Use substitution to solve this system for Y and r in terms of the other parameters.

ii) How does the equilibrium GNP depend on the marginal propensity to save?

iii) How does the equilibrium interest rate depend on the marginal propensity to save?

i) Since taxes are zero in the chapter 6 model, "no fiscal policy" means that spending is also zero (the government is neither in surplus nor deficit). That, together with $M_s = M^o$, reduces the system to

$$\begin{aligned} sY + ar &= I^o \\ mY - hr &= 0. \end{aligned}$$

This is most easily solved by substitution. The second equation yields $Y = (h/m)r$, so the first equation becomes $[(sh/m) + a]r = I^o$. Thus

$$\begin{aligned} r &= \frac{mI^o}{sh + am} \\ Y &= \frac{hI^o}{sh + am}. \end{aligned}$$

ii) The marginal propensity to save is s. Computing $\partial y/\partial s$, we find that GNP is negatively related to the marginal propensity to save. This is the well-known "paradox of thrift" in the IS-LM model.

iii) Here too, we have a negative relation between the interest rate and the marginal propensity to save.

7.12 Use Gauss-Jordan elimination in matrix form to solve the system

$$\begin{aligned} w + x + 3y - 2z &= 0 \\ 2w + 3x + 7y - 2z &= 9 \\ 3w + 5x + 13y - 9z &= 1 \\ -2w + x - z &= 0 \end{aligned}$$

We form the augmented matrix and row-reduce.

$$\begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 2 & 3 & 7 & -2 & 9 \\ 3 & 5 & 13 & -9 & 1 \\ -2 & 1 & 0 & -1 & 0 \end{pmatrix} \xrightarrow{(2)-2(1)} \begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 3 & 5 & 13 & -9 & 1 \\ -2 & 1 & 0 & -1 & 0 \end{pmatrix}$$

$$\xrightarrow{(3)-3(1)} \begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 2 & 4 & -3 & 1 \\ -2 & 1 & 0 & -1 & 0 \end{pmatrix} \xrightarrow{(4)+2(1)} \begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 2 & 4 & -3 & 1 \\ 0 & 3 & 6 & -5 & 0 \end{pmatrix}$$

$$\xrightarrow{(3)-2(2)} \begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 2 & -7 & -17 \\ 0 & 3 & 6 & -5 & 0 \end{pmatrix} \xrightarrow{(4)-3(2)} \begin{pmatrix} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 2 & -7 & -17 \\ 0 & 0 & 3 & -11 & -27 \end{pmatrix}$$

$$\xrightarrow{(1)-1(2)} \begin{pmatrix} 1 & 0 & 2 & -4 & -9 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 2 & -7 & -17 \\ 0 & 0 & 3 & -11 & -27 \end{pmatrix} \xrightarrow{(3)/2} \begin{pmatrix} 1 & 0 & 2 & -4 & -9 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 3 & -11 & -27 \end{pmatrix}$$

$$\xrightarrow{(1)-2(3)} \begin{pmatrix} 1 & 0 & 0 & 3 & 8 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 3 & -11 & -27 \end{pmatrix} \xrightarrow{(2)-(3)} \begin{pmatrix} 1 & 0 & 0 & 3 & 8 \\ 0 & 1 & 0 & 11/2 & 35/2 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 3 & -11 & -27 \end{pmatrix}$$

$$\xrightarrow{(4)-3(3)} \begin{pmatrix} 1 & 0 & 0 & 3 & 8 \\ 0 & 1 & 0 & 11/ & 35/2 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & -1/2 & -3/2 \end{pmatrix} \xrightarrow{2(4)} \begin{pmatrix} 1 & 0 & 0 & 3 & 8 \\ 0 & 1 & 0 & 11/2 & 35/2 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{(1)-3(4)} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 \\ 0 & 1 & 0 & 11/2 & 35/2 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix} \xrightarrow{(2)-11(4)/2} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix} \xrightarrow{(2)+7(4)/2} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

The solution is $(w, x, y, z) = (-1, 1, 2, 3)$.

7.18 For what values of the parameter a does the following system of equations have a solution?

$$\begin{aligned} 6x + y &= 7 \\ 3x + y &= 4 \\ -6x - 2y &= a. \end{aligned}$$

Form the augmented matrix and row-reduce.

$$\begin{pmatrix} 6 & 1 & 7 \\ 3 & 1 & 4 \\ -6 & -2 & a \end{pmatrix} \xrightarrow{(1)/6} \begin{pmatrix} 1 & 1/6 & 7/6 \\ 3 & 1 & 4 \\ -6 & -2 & a \end{pmatrix} \xrightarrow{(2)-3(1)} \begin{pmatrix} 1 & 1/6 & 7/6 \\ 0 & 1/2 & 1/2 \\ -6 & -2 & a \end{pmatrix} \xrightarrow{(3)+6(1)}$$

$$\begin{pmatrix} 1 & 1/6 & 7/6 \\ 0 & 1/2 & 1/2 \\ 0 & -1 & a+7 \end{pmatrix} \xrightarrow{2(2)} \begin{pmatrix} 1 & 1/6 & 7/6 \\ 0 & 1 & 1 \\ 0 & -1 & a+7 \end{pmatrix} \xrightarrow{(1)-(2)/6} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & a+7 \end{pmatrix} \xrightarrow{(3)+(2)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & a+8 \end{pmatrix}.$$

Only when $a + 8 = 0$ ($a = -8$) does the system have a solution. In that case, $x = 1$ and $y = 1$.