Chapter 10:

Fixed Income Analysis

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- Mortgage
- Mortgage Backed Security
- Amortization of Mortgage
  - time t interest:
  $$iM_{t-1}$$
  - time t principal payment:
  $$PMT - iM_{t-1}$$
  - time t principal:
  $$M_t = M_{t-1}(1 + i)$$
  - zero amortization (perpetuity) if
  $$PMT = iM$$

– negative amortization (graduated payment mortgage) if

$$PMT < iM$$

– positive amortization if

$$PMT > iM$$

– level payment mortgage:

$$PMT = \frac{M_t}{A(i, n)}$$

where M is the principal value and i equals contract rate divided by 12

– payment equals interest plus scheduled principal payment

- Servicing Fee

  – portion of contract mortgage rate

  – servicing involves

    * collecting monthly payments
    * forwarding payment to loan owners
    * maintaining records
    * sending out late notices
    * collecting late fees
    * administrating escrow account
    * furnishing tax information to borrowers

- Prepayments
  - payment made in excess of monthly payment
  - reduces principal outstanding
  - typically no prepayment penalty
  - most mortgages are nonassumable

- because prepayment amount and timing of cash flows is uncertain

- Conditional Prepayment Rate (CPR)
  - SMM (single monthly mortality) is the fraction of principal prepaid in a given month
  - (1-SMM) is the monthly survival rate
  - (1-CPR) is annual survival rate
  - CPR is annualized SMM:

    $$1 - CPR = (1 - SMM)^{12}$$

  - typically you are given the CPR and need to find SMM:

    $$SMM = 1 - (1 - CPR)^{1/12}$$

  - prepayment for month t

    $SMM \times (M_{t-1} - \text{Scheduled Principal Payment})$

– or

$$SMM \times (M_{t-1}(1+i) - PMT_t)$$

– scheduled payment will decline over time because of prepayments:

$$PMT_t = \frac{M_t}{A(i, n-t)}$$

where $n-t$ is number of remaining payments

– or

$$PMT_t = PMT^* \times \frac{M_t}{M_t^*}$$

where PMT^* and M_t^* are fixed payment and time t principal for a CPR $=$0% mortgage

– example 1:

 * CPR $=$ 6%

 * $M_{t-1} =$ $290M

 * scheduled payment $=$ $3M

* single monthly mortality

$$SMM = 1 - (1 - .06)^{1/12}$$
$$0.5143\%$$

* prepayment:

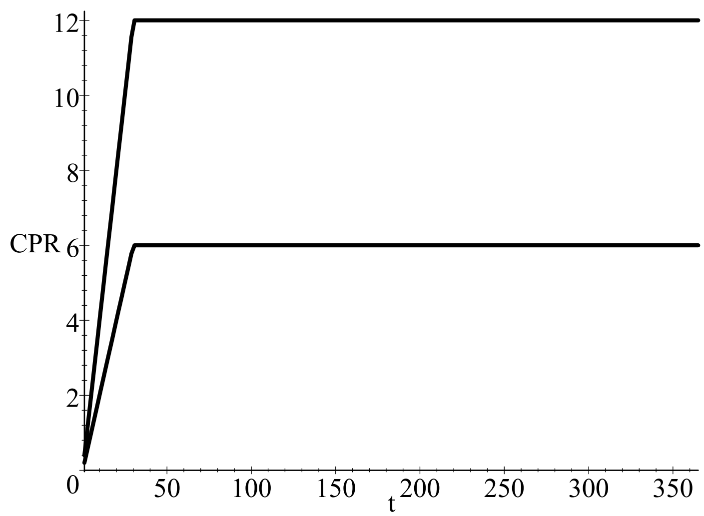
$$(287 - 3) \times .005143 = \$1.4606\text{M}$$

* example 2: see Excel spread sheet

- PSA (Public Securities Association) Prepayment Benchmark
  - seasoned mortgage:
    * issued at least 30 months prior
    * 100% PSA $\leftrightarrow$ 6% CPR
    * 200% PSA $\leftrightarrow$ 12% CPR
  - unseasoned

* 100% PSA $\leftrightarrow CPR = \begin{cases} \frac{6}{30}t & \text{for} \quad t \le 30 \\ 6 & \text{for} \quad t > 30 \end{cases}$

* 200% PSA $\leftrightarrow CPR = \begin{cases} \frac{12}{30}t & \text{for} \quad t \le 30 \\ 12 & \text{for} \quad t > 30 \end{cases}$



– based upon CPR must determine SMM to project cash flow

– CPR is a benchmark – future prepayment experience may differ

– cash flow yield: IRR based upon CPR projected cash flows

- average life:

$$\frac{1}{12}\sum_{i=1}^{T}\frac{t \times (\text{projected principal payment})}{\text{total principal}}$$

- Factor Affecting Prepayment
  - current mortgage rates
  - historical prepayment experience
  - seasonal factors
  - economic activity
  - location
  - demographic characteristics
- Prepayment model
  - predicts prepayments for given mortgage pool based upon above factors

  - dynamic model because prepayments change with underlying factors

  - model is path depedendant

  - typically use Monte Carlo Simulation

  - PSA benchmark is a static

- Contraction risk

  - caused by prepayments increasing when rates fall

  - if rates fall,

    * price compression

    * reinvestment at lower rate

- Extension risk

- caused by prepayments slowing when rates rise

- Duration
  - contraction means duration will decrease when rates fall
  - extension means duration will increase when rates increase
  - same is true of average life

- CMOs
  - sequential pay tranches
  - accrual tranche
  - floating and inverse floating tranche (pages 412 to 414)

* idea: divide up sequential tranche in to floating and inverse floating tranche

* $M \equiv$ par value of sequential tranche

* $\theta \equiv$ ratio of floating rate tranche par value to M

* $1 - \theta \equiv$ ratio of inverse floating trance par value to M

* $CR_F = a + r$

* $CR_{IF} = K - Lr$

* $i \equiv$ periodic interest rate

* problem: choose a, K, L such that total payment to floating and inverse floating tranche equals cash flow available

* cash flow to M:

$$Mi$$

* cash flow to both tranches:

$$\theta M(a + r) + (1 - \theta)M(K - Lr)$$

* let's choose K such that floating and inverse floating components cancel out:

$$\theta Mr - (1 - \theta)MLr = 0$$

* so

$$L = \frac{\theta}{1 - \theta}$$

* now choose a, K such that sum of fixed components equals available interest:

$$\theta Ma + (1 - \theta)MK = Mi$$

* so

$$K = \frac{i - \theta a}{1 - \theta}$$

* this works so long as principal (scheduled and prepaid) is directed toward higher tranches

* so bottom tranche should be used
* once principal payments are directed toward floating and inverse floating tranches, payments are divided in accordance to principal values
* so at this point fixed and floating rate instruments amortize – or return principal
* what is the nature of the prepayment risk for floating and inverse floating tranches?
* text book example
  - total tranche = \$96.5
  - floating tranche = \$72.375
  - assume annual pay at rate of 7.5%
  - floating rate has quoted margin of 50 basis points

- this could be based on corporate floaters – not tied to inverse floaters via CMO

- $\theta =$72.375/96.5 = 3/4

- so
$$L = \frac{3/4}{1 - 3/4} = 3$$

- and
$$\begin{aligned} K &= \frac{.075 - (3/4)(.005)}{1 - 3/4} \\ &= 28.5\% \end{aligned}$$

- same answers as in the text book

– PAC and support tranches

- PO and IO only strip