Quantum Mechanics II **Name** ________________

Homework # 1 Spring 2005

Due Wednesday January 19

Problem 1: Using the matrix representation for L_z and $L_\pm$, work out the commutation relations $[L_z, L_\pm] = \hbar L_\pm$. Recall

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$L_+ = \hbar \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$L_- = \hbar \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

Problem 2: The spin matrices are defined by

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

a) Check that they obey the fundamental commutation relation for angular momentum: $[S_x, S_y] = i\hbar S_z$.
b) Show that the Pauli spin martrices satisfy

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l,$$

where the indices stand for x, y, or z, and ϵ_{jlk} is the Levi-Cirita symbol: $+1$ if $jkl = 123, 231$, or 321; -1 if $jkl = 132, 213$, or 213; 0 otherwise.

Problem 3: An electron is in the spin state

$$\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}.$$

a) Determine the normalization constant A.

b) Find the expectation values of S_x, S_y, and S_z.

c) Find the "uncertainties", σ_{S_x}, σ_{S_y}, and σ_{S_z}, Problem 4:

a) Find the eigenvalues and eigenspinors of S_y

b) If you measured S_y on a particle in the general state χ, what values might you get and what is the probability of each? Check that the probabilities add up to 1.

c) If you measured S_y^2, what values might get and what are their respective probabilities?

Problem 5: Construct the spin matrices (S_x, S_y, and S_z) for a particle of spin 1. *Hint*: How many eigenstates of S_z are there? Determine the action of S_z, S_+ and S_- on each of these states. Follow the same procedures outlined in the text and in class for spin 1/2 particles.

Problem 6: An electron is at rest in an oscillating magnetic field

$$\mathbf{B} = B_0 \cos(\omega t)\hat{k},$$

where B_0 and ω are constants.

a) Construct the Hamiltonian matrix for this system.

b) The electron starts out ($t = 0$) in the spin-up state with respect to the x-axis ($\chi(t = 0) = \chi_+^x$). Determine $\chi(t)$ at any subsequent time. Becareful, this is a time-dependent Hamiltonian, so you cannot get $\chi(t)$ in the usual way from the Schrodinger equation directly.

c) Find the probability of getting $-\hbar/2$ if you measure S_x.

d) What is the minimum field (B_0) required to force a complete flip in S_x?