COMPUTATIONAL PROBLEMS: MORTGAGE MATHEMATICS

1 . Determine the monthly payments required to repay the following fixed rate loans:

   (a) $30,000 for 30 years at 12 percent.
   (b) $30,000 for 30 years at 18 percent.
   (c) $30,000 for 30 years at 6 percent.
   (d) $30,000 for 40 years at 12 percent.
   (e) $30,000 for 20 years at 12 percent.

2. Compute the mortgage balance remaining after 20 years for each of the loans in problem 1.

3. Compute the total interest expense during the twentieth year for each of the loans in problem 1.

4. Consider a variable rate mortgage on a $30,000, 30-year loan. The following rate schedule will be employed:

   Years 1 through 10: 12 percent
   Years 11 through 15: 15 percent
   Years 16 through 20: 16 percent
   Years 21 through 30: 18 percent

Determine the level end-of-the-month mortgage payments needed to retire this mortgage.

5.Determine the 1st, 10th, 15th, 150th, 359th, and 360th payment on a $30,000, 30-year graduated payment mortgage with a monthly growth rate of .25 percent (0.0025) and an annual interest rate of 15 percent.

6. Calculate the mortgage balance remaining at the end of each payment given in problem 5.

7. Using the mortgage in problem 5, determine the payment at which negative amortization stops.

8. Using the mortgage information from problem 5 once again, compute the payment at which the original mortgage balance starts declining.

9. Suppose we have a 30-year conventional fixed rate mortgage for $30,000 at a quoted rate of 12 percent. If the bank is going to earn a yield of 12.25 percent, how many points must be charged?

10. If a 30-year fixed rate mortgage at 12 percent has 7.2 discount points, what is the yield?

SOLUTIONS TO COMPUTATIONAL PROBLEMS

1. In each case, we can apply equation (4.1).

where i = k/m. Thus,

= 30,000 X 0.01029 = $308.70

(b) MP = 30,000 X 0.01507 = $452.10

(c) MP = 30,000 X 0.00500 = $150.00

(d) MP = 30,000 X 0.01008 = $302.40

(e) MP = 30,000 X 0.01101 = $330.30

Several interesting observations can be made from these solutions:

(i) An obvious observation is that as the interest rate increases so does the mortgage payment. Comparing (a) and (b), we see that as the interest rate increased by 6 percent, the mortgage payments increased by $143.40 (from $308.70 to $452.10).

(ii) A decrease in interest rate decreases the mortgage payment by a smaller amount than the same increase in interest rate increases the monthly payment That is, increases and decreases in interest rate have asymmetric effects on the monthly payments. Compare (a) to (b) with (a) to (c).

(iii) Obviously, the longer the term of the loan, the less will be the mortgage payments. Comparing (a) and (d) we see that the mortgage payment has decreased by $6.30 (from $308.70 to $302.40) with a ten-year extension on the term of the mortgage.

(iv) A decrease in the term of the loan changes the mortgage payments by a larger absolute amount than an equal increase in the term decreases it (i.e., mortgage payment). Compare (a) and (e) to (a) and (d) to see this result.

2. From equation (3.7), we have the mortgage balance remaining after the *t*th payment as

where i = k/m. Therefore,

3. From equation (4.9), we can compute the total interest expense up to any time, *t*, as

where again $i = k/m$.

Hence, interest expense during the twentieth year is

$TIE_{240} - TIE_{228}$ =$2,561.66.

Alternatively, we could calculate total interest expense during the time intervals t and t - j as

or

4. By equation (4.3), we have

5. We have, from equation (4.10),

From equation (4.11), we have for the other months:

$MPG_{10} = (MPG_1)(1 + .0025)$
= $380.33

$MPG_{15} = (379.38)(1.0025)^{14}$
= $392-87

$MPG_{150} = (379.38)(1.0025)^{149}$
= $550.36

$MPG_{359} = (379.38)(1.0025)^{358}$

= $927.44

$MPG_{360} = (379.38)(1.0025)^{359}$

= $929.76

6. Using equation (3.12), we have

7. The payment at which the amount of the mortgage payment exactly equals the interest expense is, from equation (4.14),
Substituting in our values, we have

Negative amortization will thus stop on the 217th payment.

8. Assume that the original loan amount starts declining after *tg*th payment, where *tg* is the solution to equation (4.15). That is,

By trial and error, we find *tg* to be approximately 311. Applying equation (4.11), we find the mortgage balance remaining after the 311th payment is

$$MBG_{311} = 30{,}028.78$$

and

$$MBG_{312} = 29{,}654.49$$

Thus, after the 311th payment or with the 312th payment, the original loan balance of $30,000 starts declining.

9. Applying equation (4.18), we have

Thus the lender needs to charge about four points to achieve the desired yield.

10. Applying equation (4.19), the approximate yield is

so r_a = .0106681 per month or 12.80 percent per year. Using equation (4.18), we can solve for the exact *r* by trial and error:

The exact yield is r = .010857 per month or 13.02 percent per year.

MORE COMPUTATIONAL PROBLEMS

1. For the following 30 year FRMs, compute the monthly mortgage payments, interest expenses for the 100^{th} month, 200^{th} month, and 360^{th} month?
   a) A=$200,000, k=12% b) A=$150,000, k=10% c) A=$100,000, k=8%

2. For the following 15 year FRMs, compute the monthly mortgage payments, interest expenses for the 60^{th} month, 120^{th} month, and 180^{th} month?
   a) A=$200,000, k=12% b) A=$150,000, k=10%, c) A=$100,000, k=8%

3. You take a $200,000 VRM for 30 years. The mortgage rates are set at 9% for the first 10 years, 10% for the next 10 years and 12% for the last 10 years, what is your monthly payments?

4. Compute the monthly payments for the following 15 year $100,000 VRMs with the following mortgage rates:
   a) 6% for years 1-5, 8% for years 6-10, and 10% for years 11-15.
   b) 6% for years 1-3, 8% for years 4-6, and 10% for years 7-15.
   c) 6% for years 1-2, 8% for years 3-4, and 10% for years 5-15.

5. Consider the following 30 year ARMs of the amount $100,000, compute the monthly payment for each year if the mortgage rates are:
   a) 5% in year 1, 7% in year 2, 7.5% in year 3, 8% in year 4, and then converts to a FRM at 8.5% in the beginning of the 5^{th} year.
   b) 5% in year 1, 6% in year 2, 6.5% in year 3, 7% in year 4, and then converts to a FRM at 8% in the beginning of the 5^{th} year.
   c) 5% in year 1, 7% in year 2, 8% in year 3, 9% in year 4, and then converts to a FRM at 10% in the beginning of the 5^{th} year.

6. Compute MP_1, MP_{101}, $MP_{360,}$ $MP_{60,}$ MP_{300} for the following GPMs:
   a) A=$200,000, k=9%, n=30, g=4%/year.
   b) A=$200,000, k=10%, n=30, g=3%/year.
   c) A=$200,000, k=8%, n=30, g=2%/year.

7. Consider the following 30-year term GPM (TGPM) of the amount $100,000. Find $MP_{1,}$
   MP_{50}, MP_{121}?
   a) k=10%, the monthly payments grow at 5%/year rate for 7 years then level payments afterwards.
   b) k=8%, the monthly payments grow at 3%/year rate for 5 years then level payments afterwards.

8. Consider the following 30 year GPM with constant monthly payments in each year but the

payments have constant yearly graduated increase. Compute the monthly payments for year 1, year 10, year 20, and year 30.

a) A=$100,000, APR=8%, g=3%/yr.
b) A=$150,000, APR=9%, g=2.5%/yr.

9. You take out a $150,000 mortgage at a fixed rate of 9% for 20 years. What will be your monthly payments?
10. What will be the balance remaining on the above mortgage after 8 years. Suppose you decide to refinance the mortgage after 10 years for the remaining 10 years at 8%. What would be the monthly payments on the new mortgage?

11. What will be the interest expense and the principal payment for the 61st month on both the first and the new mortgage?

12. You take out a $100,000 mortgage at 9% for 30 years. What is the mortgage balance remaining, interest expense and principal payment after 5 and 10 years? Also find the total interest expense after 10, 20 years.

13. Suppose you have the following $1,000,000 variable rate mortgage: 9% for years 1-5, 11.4% for 6-30. Calculate the MP that is fixed for the 30 years.

14. You took out a $200,000 mortgage at 9.6% for 30 years. Nine years later rates went down to 8.4% and you are thinking about refinancing for the remaining 21 years. Should you refinance if the refinancing cost is $7,000? In the above problem should you refinance if
    (a) the loan was $50,000 <u>or</u> (b) the rates went down twenty-one years later instead of nine.

15. You borrow $200,000 at 9% for 30 years with annual growth in payments of 4%. Find MP_1 MP_{101}, MP_{360}, MP_{60}, MP_{300}.

16. In the above problem find t (when the principal stops growing and positive amortization begins).

17. You are a bank officer in charge of mortgage lending. The banks has a quoted mortgage rate of 10.8% on a 30 year loan. The goal, however, is to generate a desired rate of 12%.

How many discount points should you charge? Suppose the bank decided to charge 3 discount points. What will be the true yield?

18. You took out a $180,000 30 year mortgage 12 years ago at 12%. Interest rates have declined to 9% and you wish to refinance the mortgage for the remaining 18 years. Refinancing cost is $5,000. Calculate the amount refinanced, the old and the new monthly payments. Are you better off refinancing? Calculate the MB after another 12 years, and IE, PP for the 145^{th} month on the new mortgage.

19. In the above problem, what is the (break-even) number of years you must stay in the house after refinancing in order to recover the refinancing cost?

20. You took out a $160,000 30 year mortgage at 7.8% interest. How much interest is tax deductible for (a) 8^{th} year (b) 25^{th} year.

21. You borrow a 30 year term-graduated mortgage of $75,000 from FHA. The mortgage has the following stipulation: increasing payments of 5% per year for 5 years. The interest rate is 8.4%. Calculate (a) first payment (b) 40^{th} payment (c) 100^{th} payment (d) 200^{th} payment.

22. In the above problem, calculate the mortgage balance at the end of the
    (a) 60^{th} month
    (b) 200^{th} month.

23. You secure a $120,000 30 year graduated payment mortgage with constant monthly payments (for each year) but yearly graduated payment increases of 3.6%. Interest rate is 9%. What will be your monthly payments during the (a) 2^{nd} year (b) 10^{th} year (c) 24^{th} year.

SOLUTIONS TO PROBLEMS

1. a) $MP = (200{,}000)/[(1-(1+(0.12/12))^{-360})/(0.12/12)] = 2{,}057.23$

   $MB_{99} = (200{,}000)[(1-(1+(0.12/12)^{-(360-99)}))/(1-(1+(0.12/12)^{-360})] = 190{,}397.42$

   $IE_{100} = (0.12/12)*MB_{99} = 1{,}903.97$

   $MB_{199} = 164{,}270.98$

   $IE_{200} = (0.12/12)*164{,}270.98 = 1642.71$

   $MB_{359} = 2{,}036.86$

   $IE_{360} = 20.37$

   b) $MP = 1{,}316.36$

   $MB_{99} = 139{,}854.54$

   $IE_{100} = 1{,}165.45$

   $MB_{199} = 116{,}439.66$

   $IE_{200} = 970.33$

   $MB_{359} = 1{,}305.48$

   $IE_{360} = 10.88$

   c) $MP = 733.76$

   $MB_{99} = 90{,}634.21$

   $IE_{100} = 604.23$

   $MB_{199} = 72{,}302.93$

   $IE_{200} = 482.02$

   $MB_{359} = 728.91$

   $IE_{360} = 4.86$

2. a) $MP = 2{,}400.34$

MB_{59}=168,024.77
MB_{119}= 109,215.38
MB_{179}= 2,376.57
IE_{60} = 1,680.25
IE_{120}= 1,092.15
IE_{180}= 23.77

b) MP= 1,611.91
MB_{59}= 122,565.46
MB_{119}= 76,836.63
MB_{179}= 1,598.59
IE_{60}=1,021.38
IE_{120}=640.31
IE_{180}=13.32

c) MB=955.65
MB_{59}=79,193.95
MB_{119}=47,768.46
MB_{179}= 949.32
IE_{60}=527.96
IE_{120} = 318.46
IE_{180} = 6.33

3. MP_v= A / $PVAF_v$
$PVAF_v = (1-(1+(0.09/12))^{-120})/(0.09/12) + [((1-(1+(0.10/12))^{-120})/(0.10/12))*$
$(1/(1+0.09/12)^{120})] + [((1-(1+(0.12/12))^{-120})/(0.12/12))*$
$(1/(1+(0.09/12)^{120})*(1/(1+(0.10/12)^{120})$
= 78.94 + 30.87 + 10.50 = 120.31
MP_v = 200,000/120.31 = 1,662.37

4. a) MP_v= 895.18
b) MP_v= 941.04
c) MP_v= 974.70
For this problem use the same procedure used in problem No. 3.

5. a) First year MP = 536.82
MB_{12}= 98,524.63 (use FRM formulas)
2nd year MP = 662.21
MB_{24} = 97,440.44
3rd year MP = 694.62
MB_{36} = 96,376.95
4th year MP= 726.95
MB_{48} = 95,325.69
5th year -30 year MP 1= 759.15

b) First year MP = 536.82
MB_{12} = 98,524.63
2nd year MP = 598.05
MB_{24}= 97,224.14

3^{rd} year MP = 629.06
MB_{36} = 95,957.75
4^{th} year MP = 660,01
MB_{48} = 94,715.29
5^{th} year -30^{th} year MP = 722.30

c) 1^{st} year MP = 599.55
MB_{12}= 98,771.99
2^{nd} year MP = 663.88
MB_{24} = 97,685.08
3^{rd} year MP = 729.47
MB_{36} = 96,711.02
4^{th} year MP = 796.05
MB_{48} = 95,826.48
5^{th} - 30^{th} year MP = 863.37

6. a) MP_1 = 1,075.19
$MP_{101} = 1{,}075.19 * (1+ (0.04/12))^{100}$
= 1,499.71
MP_{360} = 3,550.79
MB_{60} = 224,192.25
MB_{300} = 154,164.86

b) MP_1 = 1,331.58
MP_{101} = 1,709.25
MP_{360} = 3,263.32
MB_{60} = 218,648.82
MB_{300} =141,934.19

c) MP_1 = 1,199.81
MP_{101} = 1,417.22
MP_{360} = 2,181.47
MB_{60} = 205,639.43
MB_{300} =102,133.14

7. a) $MP_1 = (100{,}000)/[(1-((1+(0.05/12))/(1+(0.10/12)))^{360})/((0.10/12)/(0.05/12))]$
= 537.79
$MP_{50} = 537.79\ (1+(0.05/12))^{49} = 659.32$
$MB_{84} = 100{,}000\ [(1-((1+(0.05/12))/(1+(0.10/12)))^{(360-84)})/(1-((1+(0.05/12))/(1+(0.10/12)))^{360})] * (1+(0.05/12))^{84} = 124{,}656.50$
$MB_{121} = (124{,}656.50) / ((1- (1+(0.10/12))^{-276})/(0.10/12))$
= 1155.79

b) MP_1 = 537.4
MP_{50} = 607.34
MB_{60} = 106,650.92
MB_{121} = 823.15

8. a) First amortize the loan annually
$P_1 = (100{,}000)/ [(1 -((1+0.03)/(1+0.08))^{30})/(0.08 -0.03)] = 6{,}589.48$

Since the yield is 8%, the equivalent monthly compounded rate of interest is
K_{12} = 12 $[(1+0.08)^{1/12} -1]$ = 7.72084%
Monthly payment for the first year
AMP_1 = (6,589.48)/ $[((1+(0.07721/12))^{12}-1)/(0.07721/12)]$
= 529.96
Similar
P_{10}= 6,589.48 $(1.03)^9$ = 8,597.78
AMP_{10} = 8,597.78/$[((1+(0.07721/12))^{12}-1)/(0.07721/12)]$
= 691.48
P_{20} = 11,554.69
P_{30} = 15,528.54
AMP_{20} = 929.29
AMP_{30}= 1,248.89

b) P_1= 11,580.89
AMP_1 = 927.41
P_{10} = 14,462.95
AMP_{10} = 1,158.21
P_{20}= 18,513.80
AMP_{20} = 1,482.61
P_{30} = 23,699.22
AMP_{30} = 1,897.85

9. MP = A/ PVAF = A / AF = 150,000/$[(1-(1/1.0075)^{240})/0.0075]$ = 1,349.59/monthly

10. a) MB_{96} = A$[(1 -(1/(1+(k/m)))^{mn-t})/(1-(1/(1+(k/m)))^{mn})]$ = 150,000 $[(1-(1/1,0075)^{240-96})/$
$(1 - (1/1.0075)^{240})]$ = 118,589.85

b) MB_{120} = Amount refinance = 150,000 $[(1- (1/1.0075)^{120})/(1-(1/1.0075)^{240})]$ = 106,538.84
and MP on the new loan of $106,538.84 at 8% for 10 years
k/m = 0.08/12 = 0.00667
MP = 106,539 /$[(1-(1/1.00667)^{120})/0.00667]$ = 1,292.61/monthly

11. 1st mortgage: First Fund MB_{60} = 150,000$[(1-(1/1.0075)^{180})/(1-(1/1.0075)^{240})]$ = 133,060.57
IE_{61} = 0.0075 (133,060.57) = 997.95
PP_{61} = 1,349.60 - 997.95 = 351.65
2nd mortgage: $MB_{60\ new}$ = 106,539.57 $[(1-(1/1.006667)^{120-60})/(1-(1/1.00667)^{120})]$ = 63,731.76
IE_{61}= 0.00667(63,731.76) = 425.09
PP_{61}=1,292.80 - 425.09 = 867.70

12. MB_{60} = 100,000$[(1-(1/1.0075)^{360-60})/(1-(1/1.0075)^{360})]$ = 95,880.14
Similar MB_{120} = 89,429.74
MP = 804.62

IE_{61} = 0.0075(95,880.14) = 719.10 and PP_{61} = 804.62 - 719.10 = 85.52
Similar IE_{121} = 670.72 and PP_{121} = 133.90

13. $AF_1 = (1-(1/1.0075)^{60})/(0.0075) = 48.173$
$AF_2 = (1-(1/1.0095)^{300})/(0.0095) = 99.092$
$\bar{A}F = AF_1 + PVOF\ AF_2 = 48.173 + (99.092/(1.0075)^{60}) = 48.173 + 63.29 =$ 111.463
Fixed MP = A / $\bar{A}F$ = 100,000/111.463 = 897.16/month.

14. $MP_{old} = 200{,}000/[(1-(1/1.008)^{360})/0.008] = 1{,}696.32$
$MB_{108} = 200{,}000[(1-(1/1.008)^{252})/(1-(1/1.008)^{360})] = 183{,}571.55$
Now MP at 8.49% for 21 years = $183{,}571.55/[(1-(1/1.007)^{252})/0.007] = 1{,}552.71$
savings per month=143.61 and PV of savings = $143.61[(1-(1/1.007)^{252})/0.007] =$ 16,978.46
Net savings = 16,978.46 -7,000 = 9978.46 > cost. Then refinance is a good idea.

a) MP = 424.08 and PV = 4,244
MB_{108} = 45,893, MP_{new} = 388.18. Savings per month = 35.90
Net savings = 4,244 - 7,000 = -3,756. In this case do not refinance.

b) MP_{old} = 1,696.32, MB_{252} = 122,363
MP_{new} for 9 years at 8.4% = 1,618.50
PV of savings = 5,883 < 7,000. In this case do not refinance.

15. $MP_1 = 200{,}000/[(1-(1.00333/1.0075)^{360})/(0.0075-0.00333)] = 1{,}075.67$
k/m=0.0075 g =0.04/12 = 0.00333
$MP_C = MP_1(1+g)^{t-1}$
$MP_{101} = 1{,}075.67(1.00333)^{100} = 1{,}499.89$
MP_{360}= 3,548.15
$MB_{60} = [200{,}000/[(1-(1.00333/1.0075)^{300})/(1-(1.00333/1.0075)^{360})]]*(1.00333)^{60}$
= 224,160.31
Similar MB_{300} = 154,066.60

16. $T^* = (360+1) - [(Ln(0.00333/0.0075))/(Ln(1.00333/1.0075))] = 361-(-0.81193/-0.0041475)$
= 165.23 months.

17. k/m=0.108/12 =0.009 r=0.12/12=0.01 d=0.03 k=0.108
$R \approx (2(0.108) + (0.03/30))/(2(1 -0.03)) = 0.1118$ = 11.18% /year.(True yield).
$d= 1-[((1-(1/1.01)^{360})/0.01)/(1-(1/1.009)^{360})/0.009)] = 0.0888$ = 8.88 points.

18. MB_{144} = 163,567.30 Amount to be refinance
MP_{old} = 1,851.50
$MP_{new} = 163{,}567.30/(1-(1/1.0075)^{216}) = 1{,}531.72$ savings per month = 319.78
PV of savings = 34,148.
Net savings = 34,148 - 5,000 = 29,148. In this case is clear that there should be refinance.

MB_{144} on new mortgage = 163,567.30[(1-(1/1.0075)^{72})/(1-(1/1.0075)^{216})] = 84,974.86

$IE_{145\ new}$ = 0.0075(84,975.86) = 637.31

PP_{145} = 1,531.72 - 637.31 = 894.41

19. Savings/month = 319.78. Need to recover 5,000 then A=5,000 P=319.78 k/m=0.0075

n = [-Ln (1-((5,000)(0.0075/319.78)))]/(12Ln(1.0075) = 1.391 years (16.69 months) to break even.

20. Tax deductible interest for the 8th year = TIE_{96} - TIE_{84}

MP = 160,000/((1-(1/1.0065)^{360})/0.0065) = 1,151.79

TIE_{84} = 1,151.79[(84-(((1.0065)^{84} -1)/(0.0065)(1.0065)^{360})] = 84,310.87

Similar TIE $_{96}$ = 95,735.87 and tax deductible interest 95,735.87 - 84,310.87 = 11,425

Similar Tax deductible interest for the 25th year = 242,610.85 - 237,775.02 = 4,835.83

21. a) MP_1 = 75,000/[(1-(1.004167/1.007)^{360})/(0.007-0.0041670] = 333.39

b) MP_{40} = 333.39 (1.004167)^{39} = 392.09

c) After 60 months the loan becomes fixed rate and fixed MP mortgage.

MB_{60} = (75,000)* [(1-(1.004167/1.007)^{300})/(1-(1.004167/1.007)^{360})]*(1.004167)^{60}

= 86,165.98

d) MB_{100} = MP_{200} = 86,165.98/ [(1-(1/1.007)^{300})/0.007] = 688.03

22. MB_{60} = 86,165.98

MB_{200} = 86,165.98 [(1-(1/1.007)^{160})/(1-(1/1.007)^{300})] = 66,095.04

23. Using annual compounding

P_{1yr} = 120,000/[(1-(1.036/1.09)^{30})/(0.09-0.036)] = 8,284

P_2 = 8,284(1.036) = 8,582.23 This is for the 2nd year.

P_{10} = 8,284 (1.036)^{9} = 11,388.80

P_{24} = 8,284(1.036)^{23} = 18,685.92

AMP_2 annualized = Equivalent MP_{24} for the 2nd year = 8,582.20/[((1.0075)^{12} - 1)/(0.0075)

= 686.16/month.

$AMP_{10\ year}$ = 910.55

$AMP_{24\ year}$ = 1,493.96

THEORETICAL PROBLEMS: MORTGAGE MATHEMATICS

1. Show that the number of payments, t, required to exactly halve the original loan amount, under a fixed rate Mortgage, is equal to

   where $i = k/m$.

2. Suppose an individual can afford a mortgage payment of only \$MP. Determine how long it will take to repay a loan of \$A with an interest rate of k percent. Assume m compounding periods per year and a conventional fixed rate mortgage.

3. Suppose again an individual can afford a mortgage payment of only $MP. How expensive a home can the individual afford, assuming a conventional fixed rate 30-year mortgage at k percent?

4. Show that the total interest expense after the tth payment, TIE_t, on a fixed rate mortgage can be written as:

5. Show that the total interest expense on a fixed rate mortgage during a given period, t - p to t, can be written as:

6. Determine how long it would take to repay a loan under a graduated payment mortgage with an interest rate of k a loan amount of $A, and a periodic growth rate of g.

7. Show that the formula for the monthly mortgage payment for a graduated payment mortgage is exactly the same as that for the fixed rate mortgage when g = 0.

8. Suppose we have a conventional fixed rate loan but the lender believes the buyer will make exactly q(<mn) payments and will pay off the balance in the qth period. Derive the formula for the discount points to be charged in such a situation to obtain a desired yield, r.

SOLUTIONS TO THEORETICAL PROBLEMS

1. The mortgage balance outstanding, MB_t, after the *t*th payment is given by equation (4.7) as

We need to solve this expression for *t.* Rewriting, we have

Taking the logarithm of both sides and solving for *t*, we have

Since the problem calls for *MB/A = 1/2,* the result follows.

Note that this result is general and actually shows the number of payments required under a fixed rate mortgage to reach a desired level of the remaining mortgage balance.

2. We know that the mortgage payment, *MP*, from equation (4. 1) is equal to

We need to solve this expression for *n*. Rewriting, we have

It should be noted that unless 1 - (A/MP) $\times$ *(k/m) > 0* or, equivalently, *MP > Ak/m* the loan will never be repaid.

Expanding this expression, we have

where i = k/m.

Rearranging, we have

The first bracketed term is equation (4. 1) or *MP*. The second bracketed term is equal to -1. The third bracketed term is equation (4.7), the mortgage balance remaining after the *t*th payment.

Therefore,

This alternative representation of the total interest expense provides an intuitive computational method. Intuitively, the total interest is the total value of payments minus the principal paid. The term $(A - MB_t)$ *is* the principal paid; hence, the formula yields to a very intuitive result.

5. In the previous problem, we showed

It must also be true that

Our result can be obtained by subtracting those two expressions to obtain

6. From equation (4.11), the monthly payments with *i = k/m* are given as

We want to solve this expression for *n.* Rearranging, we have

Taking logs and solving for *n* yields

7. The monthly mortgage payments under a graduated payment mortgage, by equation (4.11), are

Substituting $g = 0$ and simplifying yields

This is just equation (4. 1), the formula for constant monthly payments under a fixed rate conventional mortgage.

8. There will be only q periodic payments, and after the qth payment the mortgage balance remaining is MB_q. Hence, the present value of an annuity of \$*MP* for q periods and the present value of a single sum of MB_q *at effective* yield r must be equal to $A - dA.$ Algebraically,

Substituting for *MP* from equation (4. 1) and MB_q from equation (4.7), we have

where $i = k/m.$ Solving for d we have