

## GRAPHING POLYNOMIALS

An  $n$ th degree polynomial function  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  ( $n$  a whole number,  $a_n \neq 0$ ) can be graphed using the following steps:

- 1) Find the y-intercept  $P(0) = a_0$ .
- 2) Find all the zeros of the polynomial. The zeros are the numbers  $x_0$  for which  $P(x_0) = 0$ . Graphically,  $x_0$  is an x-intercept of  $P$ . Indicate the zeros on the your graph.
- 3) Use the multiplicity of  $x_0$  to decide whether the graph of  $P$  touches or crosses the  $x$ -axis at  $x_0$ .
- 4) Determine the intervals of increasing and decreasing of  $P$ , and find all the relative extremum using the First or Second Derivative Tests. Indicate the relative extremum on the graph. Since polynomials have no points of non-differentiability, then any relative extrema will occur at stationary points.
- 5) Determine the intervals of concave up and concave down. Find and label any inflection points.
- 6) Extend your graph to the left and right by finding the end behavior of  $P$ . The end behavior is completely determined by  $\lim_{x \rightarrow \infty} a_n x^n$  and  $\lim_{x \rightarrow -\infty} a_n x^n$ .

## NOTES:

- 1) Polynomials are defined, continuous, and infinitely differentiable on the Set of Real Numbers  $(-\infty, \infty)$ . (Infinite differentiability implies the other two features).
- 2) Polynomials have no vertical asymptotes. This is because polynomials are continuous everywhere. Also, non-constant polynomials have no horizontal asymptotes because their limits at  $\pm\infty$  do not exist (as finite values).
- 3) Please keep in mind that if a function has a relative extremum, then the extremum will occur at a critical point. But the converse of this statement is **not true**. In other words, being a critical point is **necessary** but **not sufficient** condition for an extremum to occur. Existence of a critical point does not guarantee you will have an extremum there. A classic example of this is the function  $f(x) = x^3$ .  $f'(x) = 3x^2 \Rightarrow x = 0$  is a critical point but no sign change of the derivative occurs at  $x = 0$ . Therefore, there is no relative maximum or minimum of  $f$  at  $x = 0$ , or anywhere else, because  $x = 0$  is the only critical point. In fact,  $f(x) = x^3$  is increasing on  $(-\infty, \infty)$  (why?).
- 4) Since the derivative of an  $n$ th degree polynomial is a polynomial of degree  $n - 1$  and the maximum number of zeros a polynomial can have is equal to its degree, then a polynomial of degree  $n$  cannot have more than  $n - 1$  critical points. We can now conclude that an  $n$ th degree polynomial can have at most  $n - 1$  relative extrema (why?).
- 5) Using similar reasoning as in #4 above, no polynomial can have more than  $n - 2$  inflection points.