

THE FUNDAMENTAL IDENTITIES

KNOW THESE TO THE POINT WHERE IF AWAKENED FROM A DEEP SLEEP IN THE MIDDLE OF THE NIGHT, YOU WOULD BE ABLE TO RECITE THEM ALL IN NO MORE THAN TWO MINUTES!

1) Reciprocal Identities:

a) $\csc x = \frac{1}{\sin x}$ or $\sin x = \frac{1}{\csc x}$ or $(\csc x)(\sin x) = 1$ b) $\sec x = \frac{1}{\cos x}$ or $\cos x = \frac{1}{\sec x}$ or $(\sec x)(\cos x) = 1$

c) $\cot x = \frac{1}{\tan x}$ or $\tan x = \frac{1}{\cot x}$ or $(\cot x)(\tan x) = 1$

2) Quotient Identities:

a) $\tan x = \frac{\sin x}{\cos x}$ b) $\cot x = \frac{\cos x}{\sin x}$

3) Pythagorean Identities:

a) $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x = 1 - \cos^2 x$ or $\cos^2 x = 1 - \sin^2 x$

b) $\tan^2 x + 1 = \sec^2 x$ or $\tan^2 x = \sec^2 x - 1$ or $\sec^2 x - \tan^2 x = 1$

c) $1 + \cot^2 x = \csc^2 x$ or $\cot^2 x = \csc^2 x - 1$ or $\csc^2 x - \cot^2 x = 1$

4) Co-function Identities:

a) $\cos x = \sin\left(\frac{\pi}{2} - x\right)$ b) $\csc x = \sec\left(\frac{\pi}{2} - x\right)$ c) $\cot x = \tan\left(\frac{\pi}{2} - x\right)$

5) Identities for Negatives:

a) $\sin(-x) = -\sin x$ b) $\cos(-x) = \cos x$ c) $\tan(-x) = -\tan x$

[NOTE: These are sometimes referred to as the even-odd identities because they tell us that sine and tangent are odd functions, while cosine is an even function.]

6) Sum and Difference Identities

a) $\sin(x + y) = \sin x \cos y + \cos x \sin y$ b) $\sin(x - y) = \sin x \cos y - \cos x \sin y$

c) $\cos(x + y) = \cos x \cos y - \sin x \sin y$ d) $\cos(x - y) = \cos x \cos y + \sin x \sin y$

e) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ f) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

7) Double-Angle Identities:

a) $\sin 2x = 2 \sin x \cos x$ b) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$ c) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

8) Half-Angle Identities: (The sign + or - depends on the quadrant of $\frac{x}{2}$)

a) $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$ b) $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$ c) $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$