MAC 2311

FINDING ABSOLUTE EXTREMA

Here are some common cases concerning the existence of absolute extrema for a continuous function *f* on an interval *I*.

CASE ONE: $I=[a,b]$. The existence of both an absolute minimum and absolute maximum is guaranteed by the Extreme Value Theorem (**E**xtremely **V**aluable **T**heorem).

CASE TWO: *p* is an even degree polynomial. Then $\lim_{x\to\pm\infty} p(x)=\infty$ or $\lim_{x\to\pm\infty} p(x)=-\infty$. Therefore, in the former case, *p* will have an absolute minimum on $(-\infty,\infty)$ but no absolute maximum and in the latter, an absolute maximum on $(-\infty,\infty)$ but no absolute minimum.

CASE THREE: *f* is an odd degree polynomial. Here if the leading coefficient is positive, then $\lim_{x\to\infty} p(x)=\infty$ but $\lim_{x\to-\infty} p(x)=-\infty$ and if the leading coefficient is negative, then $\lim_{x\to\infty} p(x)=-\infty$ but $\lim_{x\to-\infty} p(x)=\infty$. Therefore, it is impossible for any odd degree polynomial *p* to have any absolute extrema on $(-\infty,\infty)$.

CASE FOUR: $\lim_{x\to a^+} f(x)=\infty$ or $\lim_{x\to b^-} f(x)=\infty$ and $f(c)$ is only relative minimum of *f* on the interval (a,b). Then $f(c)$ must be the absolute minimum of *f* on (a,b). EX) $f(0)=\frac{1}{4}$ is the absolute minimum of $f(x)=\frac{1}{4-x^2}$ on $(-1,2)$. Verify that! Also check $\lim_{x\to 2^-} f(x)=\infty$.

CASE FIVE: $\lim_{x\to a^+} f(x)=-\infty$ or $\lim_{x\to b^-} f(x)=-\infty$ and $f(c)$ is the only relative maximum of *f* on the interval (a,b). Then $f(c)$ must be the absolute maximum of *f* on (a,b). EX) $f(0)=-\frac{1}{4}$ is the absolute maximum of $f(x)=-\frac{1}{4-x^2}$ on $(-2,1)$. Verify! Also check $\lim_{x\to -2^+} f(x)=\infty$.

CASE SIX: Let *L* be a real number. If $f(c)$ is only relative extremum of *f* on an interval *I*, then $f(c)$ is the only absolute extremum of *f* on *I*. EX) $f(0)=1$ is the only relative extremum (in this case a relative maximum) of $f(x)=\frac{1}{x^2+1}$ on $[-20,7)$. Verify!